MOTION OF A SHIP AT THE FREE SURFACE

by

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United States
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THESIS

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ABSTRACT

Forced harmonic heaving motion of a ship at the free surface of an inviscid incompressible fluid is analyzed. Added mass and damping are determined using finite element techniques. Iso-parametric elements with curved boundaries allow accurate representation of the hull shape. A computer program is developed and results are found to agree closely with previously obtained theoretical and experimental results.

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I. INTRODUCTION

The motion of a ship at the free surface is dependent upon the hydrodynamic forces acting on the hull. These forces are conventionally separated into inertial, damping, and buoyant forces. The present study is confined to an evaluation, by the finite element method, of the steady-state inertial and damping forces on a cylindrical hull undergoing vertical (heaving) harmonic oscillation. The water is treated as inviscid and incompressible and a linearized boundary condition at the free surface is applied in view of the smallness of the motion. Water depth is large, but finite, and the width of the fluid region is made effectively infinite through use of a radiation (non-reflecting) boundary condition.

II. DEVELOPMENT OF THE PROBLEM

A. GOVERNING EQUATION

In the present problem the dependent variable is chosen to be the "over-pressure" (excess over static pressure) p. The finite element technique replaces the governing partial differential equation and boundary conditions by a set of simultaneous ordinary differential equations with time as the independent variable and the pressures at a large number of discrete points (nodes) as the dependent variable. The details of this transformation are given by Zienkiewicz and Newton [1]*. The resulting equations are

$$[Q_o]\{\ddot{p}\} + [D]\{\dot{p}\} + [H]\{p\} = -\rho[L]^T\{\ddot{\delta}\} \tag{1}$$

where

$[H]$ = fluid "stiffness" matrix, n x n

$[D]$ = radiation damping matrix, n x n

$[Q_o]$ = surface wave matrix, n x n

$[L]^T$ = ship-fluid interface matrix, n x k

ρ = fluid density, scalar

$\{p\}$ = vector of nodal pressures, n x 1

$\{\delta\}$ = vector of normal components of nodal displacement along ship-fluid interface, k x 1

n = number of fluid nodes

k = number of structure nodes on ship-fluid interface

and dots indicate time differentiation, i.e., $\frac{\partial \delta}{\partial t} = \dot{\delta}$.

For a steady state solution at circular frequency ω, the time variation of p may be represented as

---

*Numbers in square brackets designate references listed on page 56.

$$p(x,y,t) = p(x,y)e^{i\omega t}$$

With this and a similar substitution for δ , Eq. 1 becomes

$$\left(-\omega^2[Q_o] + i\omega[D] + [H]\right)\{p\} = \rho\omega^2[L]^T\{\delta\} \quad (2)$$

where the elements of Q_o, D, H, and δ are all real constants and the elements of p are complex constants.

B. ISO-PARAMETRIC ELEMENTS

The finite element technique requires subdivision of the region considered into a set of sub-regions (elements). For the present problem the two-dimensional parabolic iso-parametric element is used. A full account of this family of elements is given by Zienkiewicz, et al., in Ref. 2. Attention is confined here to the parabolic element.

Consider the rectangle of Fig. 1. The element has four corner nodes and four mid-side nodes, numbered as shown. In the local (ξ , η) coordinate system the edges are described by η = -1, ξ = 1, etc. The

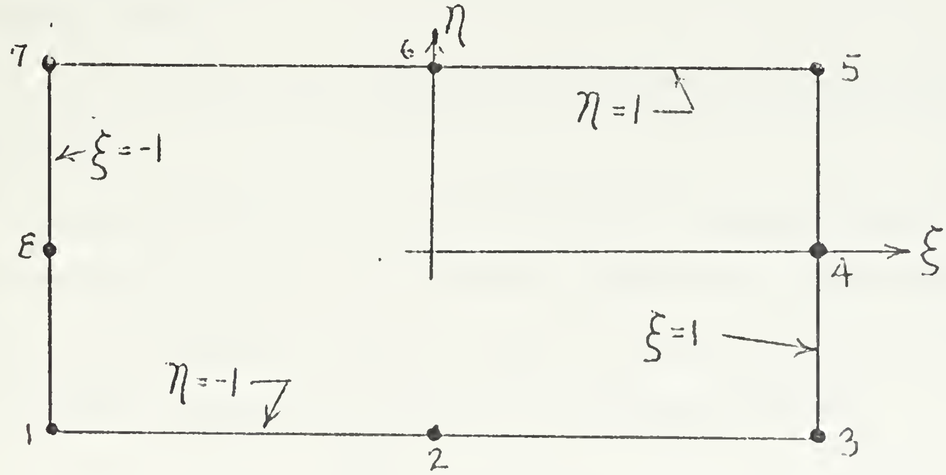


Fig. 1. Parabolic Element, Local Coordinates

dependent variable p at any point (ξ , η) of the element is given

by

$$p = \sum_{i=1}^{8} N_i \, p_i \tag{3}$$

where p_i = pressure at node i,

and

$N_i = \frac{1}{4}(1 + \xi\xi_i)(1 + \eta\eta_i)(\xi\xi_i + \eta\eta_i - 1)$ at corner nodes

or

$N_i = \frac{1}{2}(1 - \xi^2)(1 + \eta\eta_i)$ at mid-side nodes when $\xi_i = 0$

or

$N_i = \frac{1}{2}(1 - \eta^2)(1 + \xi\xi_i)$ at mid-side nodes when $\eta_i = 0$

in which ξ_i = value of ξ at ith node

η_i = value of η at ith node

The interpolation functions N_i, hereafter called "shape" functions, satisfy two essential criteria:

1. If all p_i are equal, p will have this same constant value for the entire element;

2. The value of p along any edge depends only on the p_i at nodes on that edge.

The most useful property of this iso-parametric element is that the same shape functions may be used to map the element into a curvilinear quadrilateral whose edges are parabolic arcs. Thus, if (x_i, y_i) are the Cartesian (global) coordinates of node i, the mapping function

$$(x, y) = \sum_{i=1}^{8} N_i \, (x_i, y_i) \tag{4}$$

will transform the rectangle of Fig. 1 into the form shown in Fig. 2.

Note that the element shape is determined uniquely by specifying the eight pairs of nodal coordinates (x_i, y_i).

For the present application the parabolic element provides obvious advantages in accurately representing the hull shape along the ship-fluid

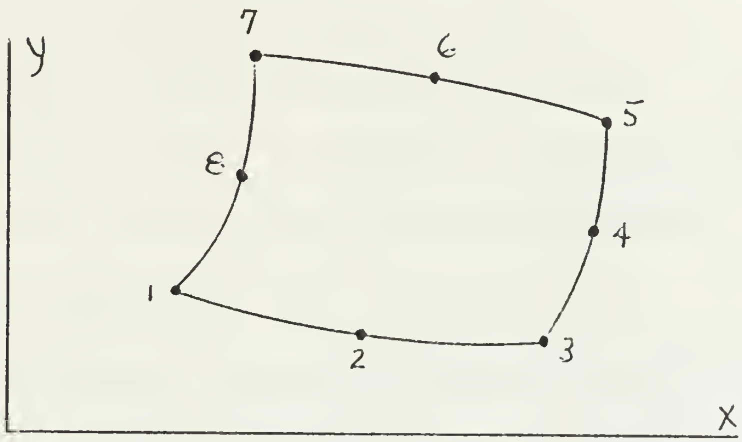


Fig. 2 Parabolic Element, Global Coordinates

interface. Elsewhere, data input is simplified if element edges are straight. It is, nevertheless, computationally economical to employ the eight-noded quadrilaterals throughout the fluid region.

C. "STIFFNESS" MATRIX H.

Matrix H is called the "stiffness" matrix by analogy with the corresponding matrix in elasticity problems. Each element contributes an 8 x 8 submatrix to H. It is shown in Ref. 1 that an individual element of this submatrix is given by

$$h_{ij} = \int_{R_e} \left[\frac{\partial N_i}{\partial x} \cdot \frac{\partial N_j}{\partial x} + \frac{\partial N_i}{\partial y} \cdot \frac{\partial N_j}{\partial y} \right] dR \qquad (5)$$

where the integration extends over the volume R_e of the element. Because the shape functions N_i are known as functions of ξ and η, not x and y, it is necessary to transform this integral to ξ, η coordinates. It is impractical to perform this transformation in algebraic form, but it is wholly feasible to do numerically and then perform the integration by Gaussian numerical quadrature. Details are given in Ref. 3.

The final step is to add the contribution of each h_{ij} to the matrix H using the global node numbers corresponding to the element nodes. This is a standard assembly process of the finite element method.

D. SURFACE WAVE CONDITION

A linearized boundary condition is imposed along the free surface of the water. The Q_o matrix has non-zero terms only at the surface nodes. For example, if the element drawn in Fig. 1 were at the surface, only nodes 5, 6, and 7 would make contributions. An individual contribution to the Q_o matrix from a single element as developed in Ref. 1 is

$$q_{o_{ij}} = 1/g \int_{S_e} N_i N_j \, dS \qquad (6)$$

where the integration extends over the free surface S_e of the element and g is the acceleration of gravity.

E. RADIATION BOUNDARY CONDITION

In open water a ship would never receive reflections of waves which originate at that ship. These waves travel away from the ship dissipating energy until they die away. To represent this effect a radiation (non-reflecting) boundary is introduced. The returning wave is mathematically excluded at this boundary. This has been done in Ref. 1 and the result is the $[D]\{\dot{p}\}$ term of Eq. 1. As in the surface wave condition, only

those nodes which are on the radiation boundary make contributions. The matrix elements are

$$d_{ij} = 1/c \int_{S_d} N_i N_j \, dS \tag{7}$$

in which

c = surface wave velocity

S_d = radiation boundary of finite element surface

This condition was conceived for acoustic waves in which c was the speed of sound. It was found that, if the surface wave velocity was used instead and the boundary was placed sufficiently far from the hull, a non-reflecting boundary condition could be imposed for surface waves. The velocity is different for each depth and frequency and is calculated as follows (Ref. [4])

$$C = g/\omega \tanh\left(\frac{\omega d}{c}\right) \tag{8}$$

where

ω = frequency

g = acceleration of gravity

d = water depth

The above lends itself easily to an iteration scheme on the computer.

F. SHIP-FLUID INTERFACE

The force vector of fluid pressure on the ship hull is shown in Ref. 1 to be representable as $[L]\{p\}$ where L is a k x n matrix. The individual force components act at the interface nodes in the direction of the local normal to the interface. In the present problem the hull is rigid and its sole motion is vertical translation. Corresponding to the upward displacement,

the resulting generalized force would have a single component and the matrix [L] would be replaced by a 1 x n row vector $\{V\}^T$. The contribution of a single finite element along the interface may be expressed as

$$f_{\text{element}} = \int_{S_h} p \cos\theta \, dS \tag{9}$$

where θ is the angle between the normal to the ship-fluid interface and the vertical. Integration extends over the element interface S_h. If the element thickness (z direction) is b, then

$$\cos\theta \, dS = b\,dx = b \frac{dx}{d\xi} d\xi \tag{10}$$

Using Eqs. 10, 3, and 4, Eq. 9 becomes

$$f_{\text{element}} = b \int_{-1}^{1} \left(\sum_j \frac{\partial N_j}{\partial \xi} x_j \right) \left(\sum_i N_i p_i \right) d\xi = \{V\}^T_{\text{element}} \{p\}_{\text{element}} \tag{11}$$

Thus, for element node i

$$V_i = b \int_{-1}^{1} \left(\sum_j \frac{\partial N_j}{\partial \xi} x_j \right) N_i \, d\xi \tag{12}$$

Using the assembled vector $\{V\}$, which has non-zero elements only for ship-fluid interface nodes, the right-hand side of Eq. 2 becomes

$$\rho \omega^2 \{V\} \Delta$$

where Δ represents the displacement amplitude. Making this substitution the form of Eq. 2 used in computation becomes

$$\left(-\omega^2 [Q_0] + i\,\omega [D] + [H] \right) \{p\} = \rho \omega^2 \{V\} \Delta \tag{13}$$

III. DISCUSSION OF RESULTS

A. REGION REPRESENTATION

Ship's hulls normally have a longitudinal (vertical) plane of symmetry. Accordingly, the ship and fluid region can be adequately modeled by considering only half of the hull and one radiation boundary. It is necessary to make the region semi-width wide enough that the high local pressure gradients near the ship-fluid interface have little effect at the radiation boundary. This can be considered accomplished when increasing the width (i.e., moving the radiation boundary farther away from the hull) does not appreciably change problem results. A one percent change in added mass and damping was considered satisfactory. By making the depth of the fluid region at least one-half the surface wave wavelength, the effects of infinite depth are closely simulated.

Once the overall region size has been decided upon, the region must be subdivided into finite elements. In the vicinity of the ship-fluid interface, elements must be small in order to match closely the hull shape and accommodate the high local pressure gradients. The elements may become larger as one moves away from the interface area. Element heights may be graded, increasing with depth. Element widths may be larger as the radiation boundary is approached, but a useable upper limit seems to be about three-eights of the surface wavelength. If an element exceeds this width, the wave-form is no longer adequately represented and accuracy suffers. Radical geometry for individual elements should be avoided whenever possible. Sample meshes used to test the program are presented in Appendix B.

Various methods were used to determine if problem results were correct. The best method, of course, was comparison with theory and previously

reported experimental results. Other checks were available also. An energy check was built into the program (see Appendix A) and when average power input equaled average power output confidence was gained in the results. Inspection of the pressure phase angle was useful. Beyond about three half-beam widths away from the hull, the phase angle should be nearly constant along any vertical line and should vary linearly with x along a horizontal line.

B. SEMI-CYLINDRICAL HULL

The first tests of the program used various meshes and a semi-cylindrical hull with a radius of ten feet. The computed results showed general agreement with Ursell's theoretic values [5] for a cylinder at the surface of a fluid region with infinite depth. The computed values, along with the theoretic values, are presented in Table I.* The mesh numbers refer to the mesh numbers in Appendix B. The first mesh produced poor results and did not adequately model the area. The energy check indicated that the results were poor and along verticals the phase angle was not even close to being constant. It appears that the radiation boundary was not far enough away from the ship, because, by simply increasing the region width with the addition of extra columns of elements (meshes 2, 3, and 4), the results were greatly improved. Meshes 2, 3, and 4 gave increasingly better results as the mesh was refined and appear to be converging upon the theoretic values. As the frequency is increased, the region depth more closely represents infinite depth (wavelength decreases, therefore less region

*Ursell's values were read from a graph presented in Ref. [7].

TABLE I

Added Mass and Damping for Semi-Cylindrical Hull

δ	Mesh 1	Mesh 2	Mesh 3	Mesh 4	Mesh 5	Theoretical Values
.311	2.10	.846	.843	.841	1.09	1.25
	.645	1.87	1.85	1.85	1.64	1.7
.477	1.92	.809	.850	.848	1.01	1.1
	.691	1.47	1.45	1.44	1.34	1.4
.609	1.74	.830	.863	.867	.950	1.0
	.671	1.05	1.13	1.13	1.07	1.1
.795	1.59	1.07	.896	.896	.933	0.9
	.591	.848	.871	.882	.828	0.8
1.01	1.48	.753	.950	.939	.958	0.9
	.475	.911	.673	.655	.638	0.7
1.24	1.44	.951	.975	.977	.965	0.95
	.334	.266	.490	.510	.465	0.5

*Notes for Table I

1. δ is non-dimensional frequency, $\delta = \frac{r\omega^2}{g}$

2. For each frequency, the first value is added mass, second is damping as defined in Ref. 7.

3. Theoretical values are from curves, Figs. 15 and 16, Ref. 7.

4. Mesh 1: 50 feet wide, 30 feet deep, 16 elements, 67 nodes.

5. Mesh 2: 90 feet wide, 30 feet deep, 19 elements, 78 nodes.

6. Mesh 3: 70 feet wide, 30 feet deep, 19 elements, 78 nodes.

7. Mesh 4: 90 feet wide, 30 feet deep, 22 elements, 89 nodes.

8. Mesh 5: 70 feet wide, 55 feet deep, 24 elements, 95 nodes.

depth is required to reach one-half wavelength). With increasing frequency, however, element width imposes an upper limit. Mesh 5 was almost twice as deep as the previous meshes and thus was able to represent infinite depth at lower frequencies. An improvement of the results, especially at lower frequencies, was noted. As expected, the phase angles along a vertical are constant at lower frequencies, but tend to vary a little as frequency increases. Likewise, the energy check gave six-digit agreement at lower frequencies with deterioration to three digits as omega increased.

C. FULL FORM HULL

The full form hull of Ref. 6 was modeled and tested using two different meshes (meshes 6 and 7). The computed results of wave amplitude, heave force, and force phase angle showed excellent agreement, using either mesh, with theoretical results due to Porter and reported in Ref. 6. These values are presented in Figs. 3, 4, and 5. Again, at higher frequencies, the energy check and phase angles of the pressures along a vertical shows signs of deterioration. The finer elements of mesh 7 did not seem markedly to improve the results over mesh 6. However, mesh 7 was taken to higher frequencies than mesh 6 and it is felt that improvement would have been noted at these higher frequencies. At lower frequencies mesh 6 produced the best results, which was expected because of the greater depth of the fluid region of mesh 6.

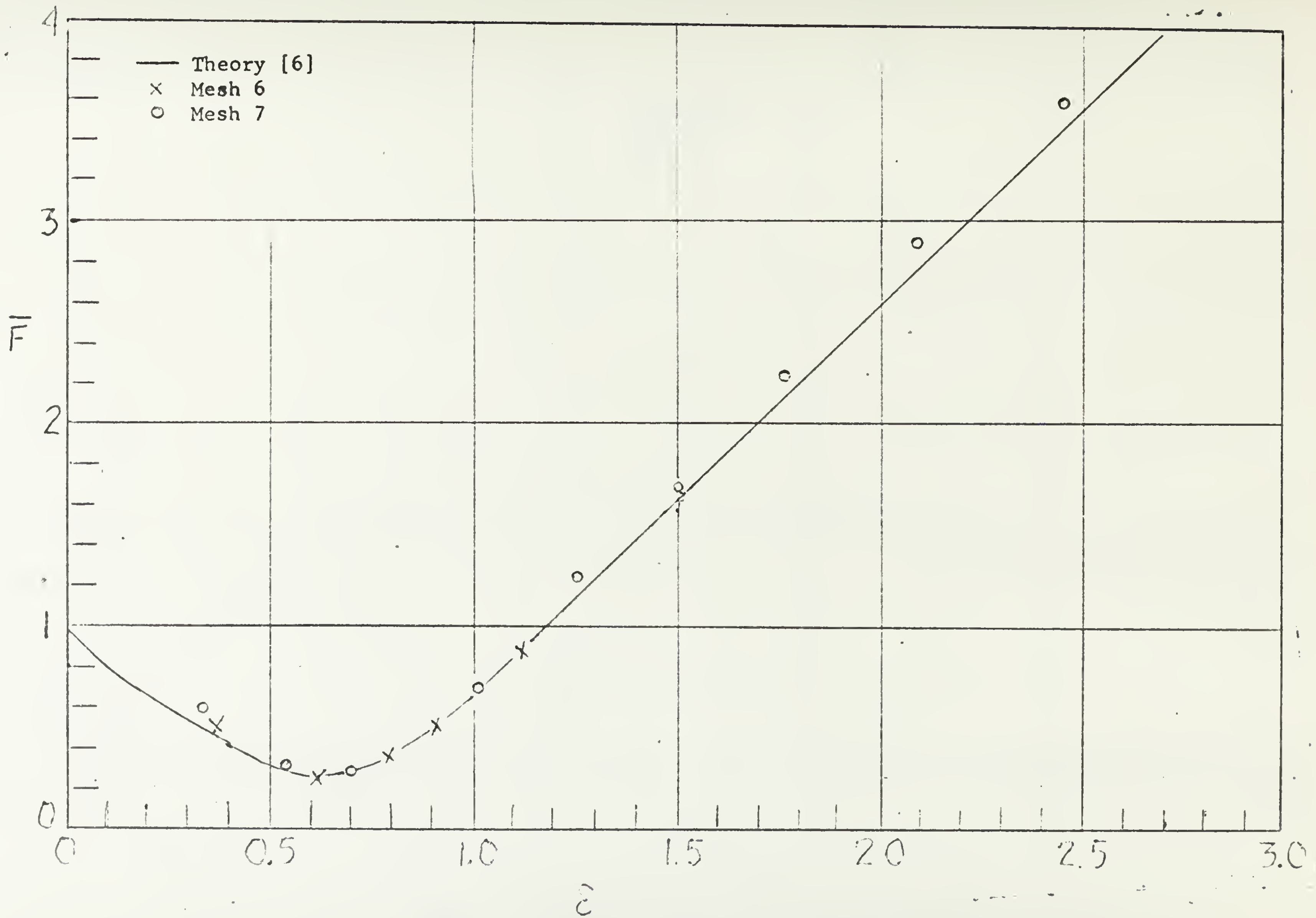


Fig. 3. Heave Force, $\overline{F}$, vs Non-dimensional Frequency, ε.

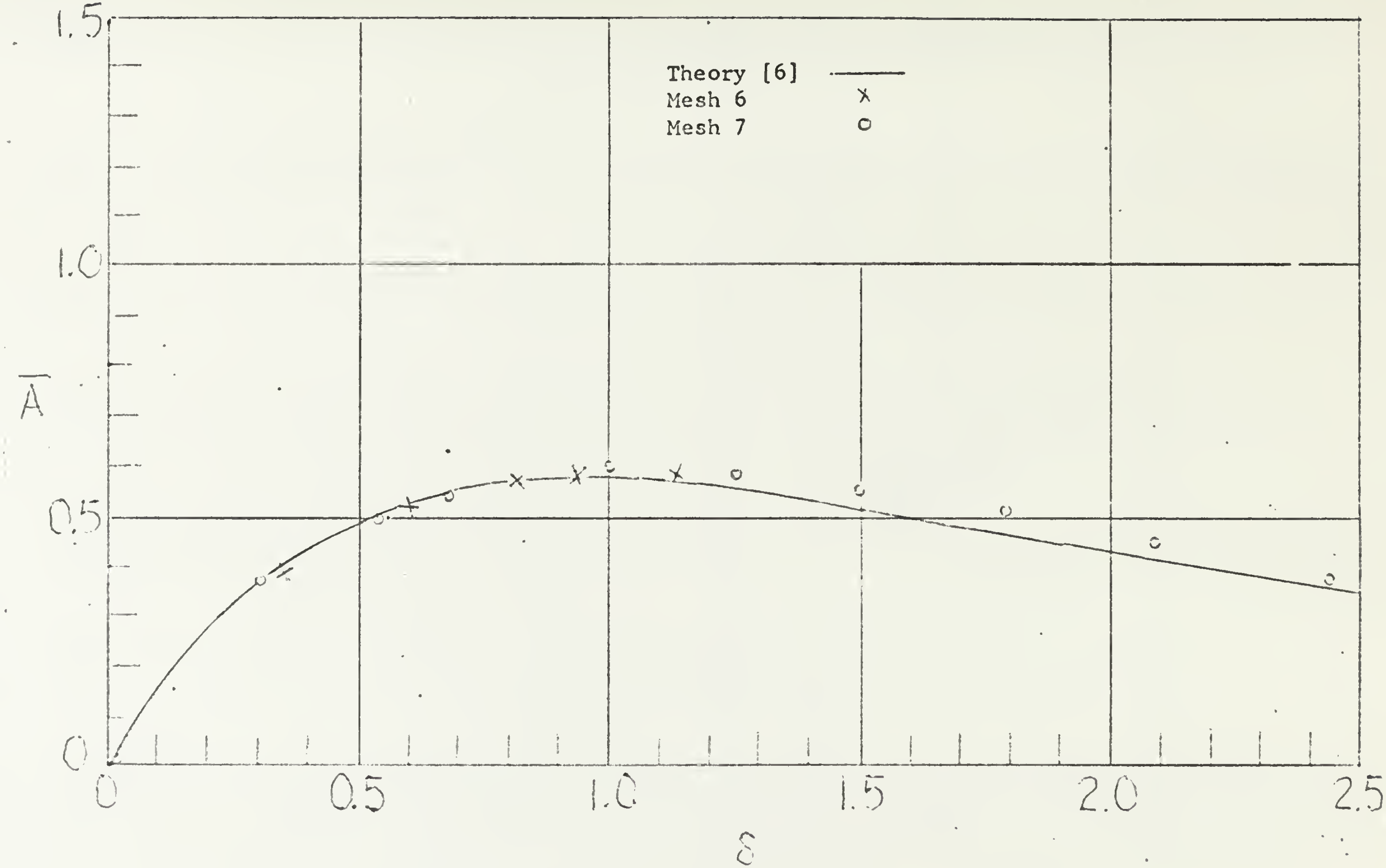


Fig. 4. Wave Amplitude, $\bar{A}$, vs Non-dimensional Frequency, δ.

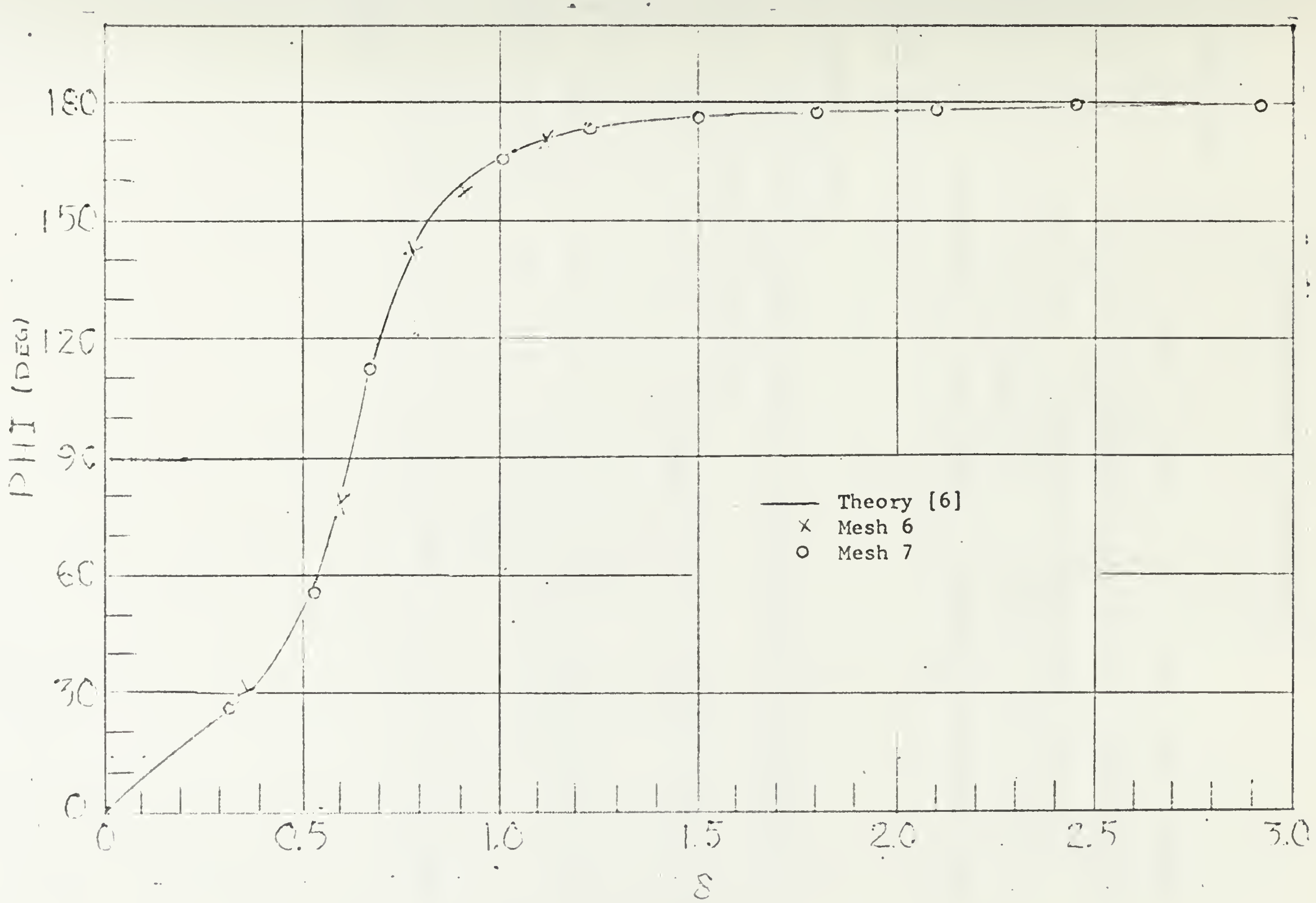


Fig. 5. Force Phase Angle, PHI, vs Non-dimensional Frequency, δ.

IV. CONCLUSIONS AND RECOMMENDATIONS

A. CONCLUSIONS

The finite element technique provides an accurate and reliable method for solution of the problem of hydrodynamic added mass and damping effects due to small-amplitude waves. A program has been developed which will solve this problem for heave motion only. For the hull forms studied, four or five iso-parametric elements along the interface adequately represent the ship's hull.

B. RECOMMENDATIONS

1. Region size and shape should be systematically investigated to determine the effects on accuracy.

2. Element size, shape, and arrangement should be studied further for greater accuracy and efficiency of computations.

3. Other hull forms should be investigated and results compared with analytic solutions. Three other hull forms are given in Ref. 6.

4. Finite depth effects should be studied.

5. The computer program should be extended to include roll and sway motions.

APPENDIX A

COMPUTER PROGRAM AND SOLUTION TECHNIQUES

A. GENERAL

The program was devised using standard FORTRAN IV language in double precision arithmetic. The testing of the program was accomplished on the IBM 360/67 digital computer installed at the Naval Postgraduate School. The main program simply calls subroutines in order. The actual program is found starting on page 35.

B. INPUT SUBROUTINE

The INPUT subroutine provides for the reading in of all necessary data and calculates certain constant problem parameters. A title of up to eighty spaces (one card) is first read in. The total number of nodes, total number of elements, and total number of "specified" nodes are read in. Next, element numbers followed by global node numbers for the local node numbers 1-8 are read in. In this process the numbers of the specified nodes or nodes at which the x and y coordinates are to be read in (generally corner nodes) are given a positive sign, while the nodes at which x and y coordinates will be calculated (mid-side nodes) are given negative node numbers. Next to be read in are the x and y coordinates of the specified nodes. These are the corner nodes of each element plus any mid-side nodes along a curved boundary (in this case the ship-fluid interface). The remaining nodes' x and y coordinates are calculated by placing them midway along a straight line connecting two adjacent nodes. These nodes are recognized by the computer by their negative sign. The total number of surface wave boundary nodes, radiation damping boundary nodes, and ship-fluid interface nodes are

read in as well as the sequence of node numbers along each of these boundary segments. Finally, water density, gravitational acceleration, and x coordinate of the ship's centerline are read in.

From the above information, the program calculates the width and depth of the region, the ship's half-beam and draft, and, using a modification of Simpson's rule, the area of the hull cross-section in the x-y plane. From these, the ship mass per unit length and vertical (buoyant) stiffness per unit length are calculated. Finally the starting value of omega, number of different values of omega to be run, and the incremental change in omega are read in. All input data and results of the above calculations are printed.

C. Q8STF SUBROUTINE

Subroutine Q8STF calculates the symmetric matrix H. It does this by calculating each element submatrix and then assembling these to form the H matrix. The calculation is based on Eq. 5, transformed to ξ , η coordinates, and uses four-point Gauss quadrature.

D. SUBROUTINES INTFAC, SURWAV, and RADAMP

Subroutines INTFAC, SURWAV, and RADAMP calculate the V, Q_o, and D matrices respectively. Subroutine INTFAC calculates the coupling vector V by moving across the ship-fluid interface by elements. It calculates the three V components at that element using three-point Gauss quadrature and then adds these values to the total V vector.

SURWAV and RADAMP are analogous except for constant factors (see Eqs. 6 and 7) and SURWAV integrates along ξ and RADAMP along η . Three-point Gauss quadrature is again used in these subroutines to compute element contributions to Q_o and D. For each element, these are symmetric 3 x 3 sub-matrices. To conserve computer storage space, the symmetric

matrices Q_o and D are stored as three vectors. These three vectors are the principal diagonal and two diagonals immediately above.

E. SUBROUTINES WAVEL and CLASS

Subroutine WAVEL is called in ELIM3 subroutine to calculate the wave velocity for each omega so that the radiation damping matrix may be multiplied by 1/c. It uses an iteration to calculate c from Eq. 8. When the iteration produces a value of c which is within .0001 percent of the previous value, an average of these two is accepted as the final value.

Subroutine CLASS classifies each node as to type for use in ELIM1, ELIM2, ELIM3, and BACSUB. The classes are:

4. All radiation boundary nodes;
3. Surface wave nodes not in 4;
2. Ship-fluid interface nodes not in 3;
1. All remaining nodes.

F. SUBROUTINES ELIM1, ELIM2, and ELIM3

The actual computational problem consists of solving n simultaneous linear equations in n unknown values of complex pressure amplitude. The coefficients of the pressure vector are formed from the matrices H, Q_o, and D. The Q_o and D matrices are sparse with contributions coming only from nodes along the free surface and radiation boundary.

Equations for Type 1 nodes do not contain ω and the corresponding variables may be eliminated initially by Gaussian elimination. If multiplication by the factor $\rho\omega^2$ on the right-hand side of Eq. 13 is deferred, the Type 2 nodes may also be included in this elimination. These operations are performed in ELIM1.

In ELIM2 the portion of H for Type 3 and Type 4 nodes is transferred to a new location and called HR. The corresponding portion of the right-hand

side is also transferred, being multiplied by $\rho\omega^2$ in the process. The contribution of $-\omega^2[Q_o]$ is added to HR and the Type 3 variables are then eliminated.

In ELIM3 a third coefficient matrix ZHR is assembled for the remaining variables (Type 4). The elements of this matrix are complex with real part coming from HR and imaginary part being $i\omega[D]$. This routine calls sub-routine SOLUN to solve the Type 4 equations.

G. SUBROUTINES SOLUN and BACSUB

Subroutine SOLUN solves the final reduced form of the simultaneous equations using Gaussian elimination and back-substitution procedures. The result is a complex pressure vector along the radiation boundary.

Subroutine BACSUB uses simple back-substitution procedures to find the remaining components of the pressure vector, finding the portion along the free surface first, followed by the Type 1 and Type 2 nodes. Next, added mass and damping are computed. If the element vertical force contributions (Eq. 9) are summed, the net force F is given by

$$F = \sum f_{element} = \{V\}^T\{p\} \tag{14}$$

The real part of F is contributed by inertia force on the added mass and the imaginary part of F is a damping force. These two components, with reversed algebraic signs, are normalized by dividing by the product of ship mass and acceleration amplitude to give dimensionless measures of added mass and damping.

An energy check is also made. Average input power is

$$\dot{E}_{in} = \tfrac{1}{2}\,\omega\,\Delta\,\mathrm{Im}\,F \tag{15}$$

The average output power (at the radiation boundary) is

$$\dot{E}_{out} = \frac{1}{2\rho c} \{p_{amp}\}^T [D] \{p_{amp}\} \tag{16}$$

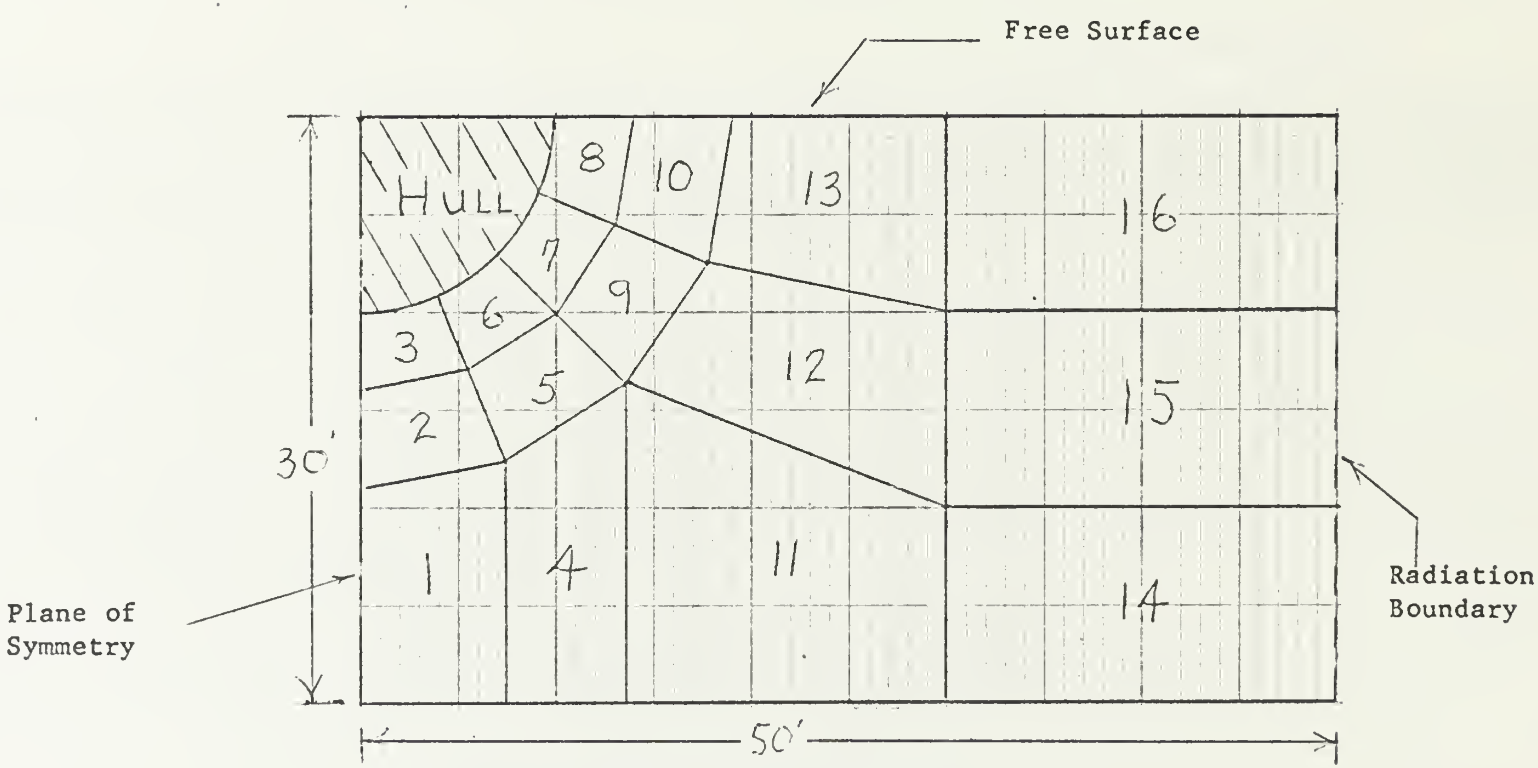
where $\{p_{amp}\}$ is a vector of nodal pressure amplitudes (absolute values) and only the radiation boundary nodes contribute to the sum.

So that a direct comparison could be made with previous results reported by Paulling and Richardson [6], non-dimensional frequency, heave force, wave height ratio, and force phase angle conforming to their definitions were calculated. Finally the value of omega is incremented by a desired amount and the problem is run over again starting at ELIM2.

APPENDIX B

MESHES

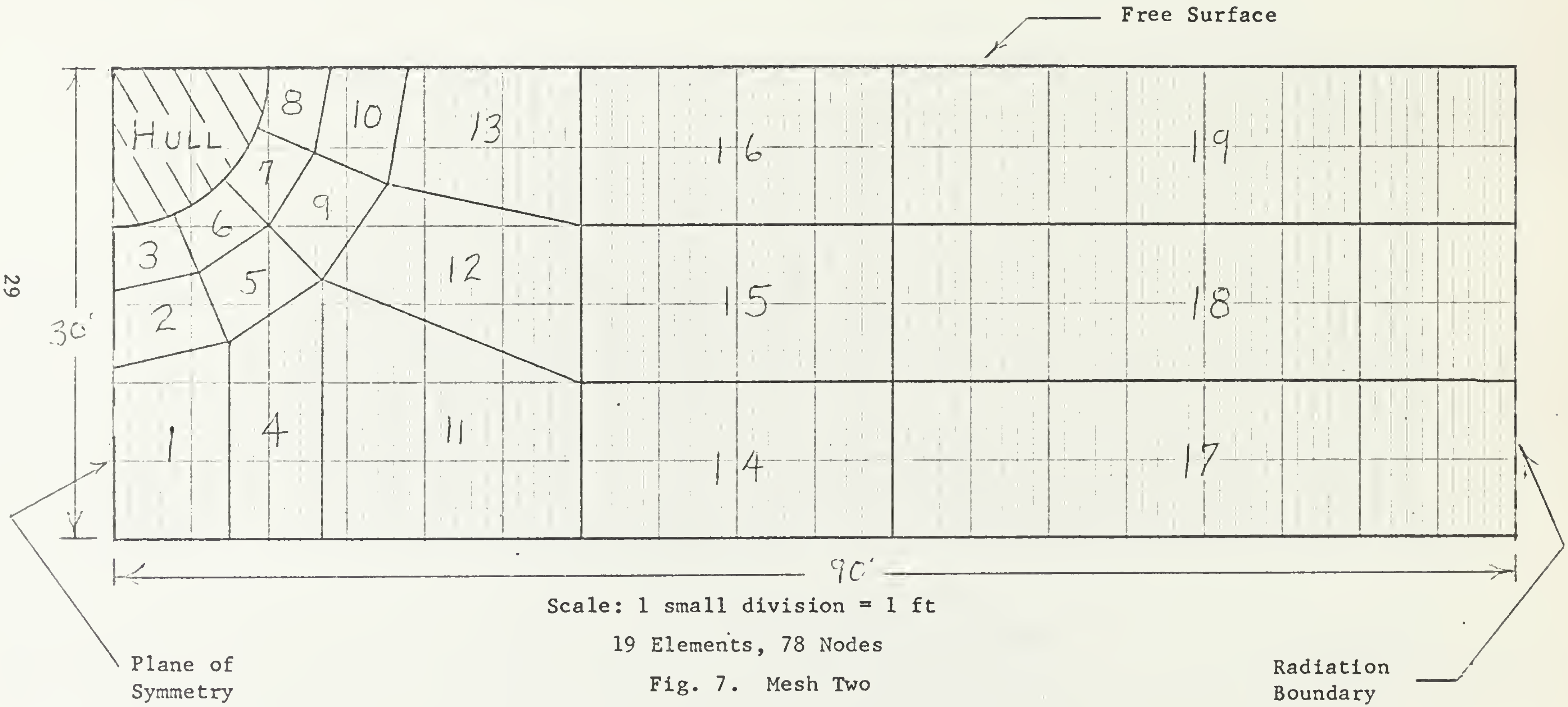
The various meshes used in testing the computer program are presented in this Appendix. Meshes 1-5 have a semi-cylinder for a hull. Mesh 1 was the basic mesh used and meshes 2-5 are simply refinements thereof, or additions thereto. Meshes 6 and 7 have a full form hull. All meshes are drawn to the same scale of one small division representing one foot. Elements are numbered, however, nodes are not.



Scale: 1 small division = 1 ft

16 Elements, 67 Nodes

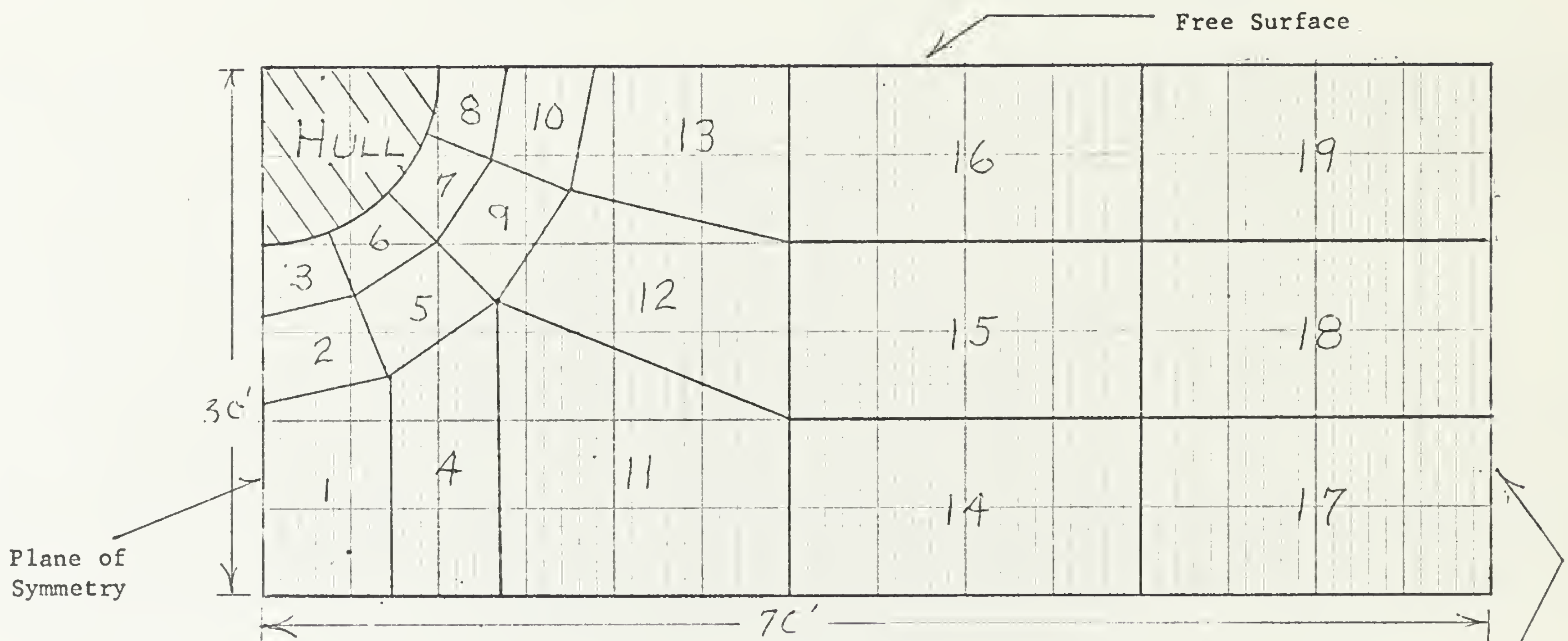
Fig. 6. Mesh One



Scale: 1 small division = 1 ft

19 Elements, 78 Nodes

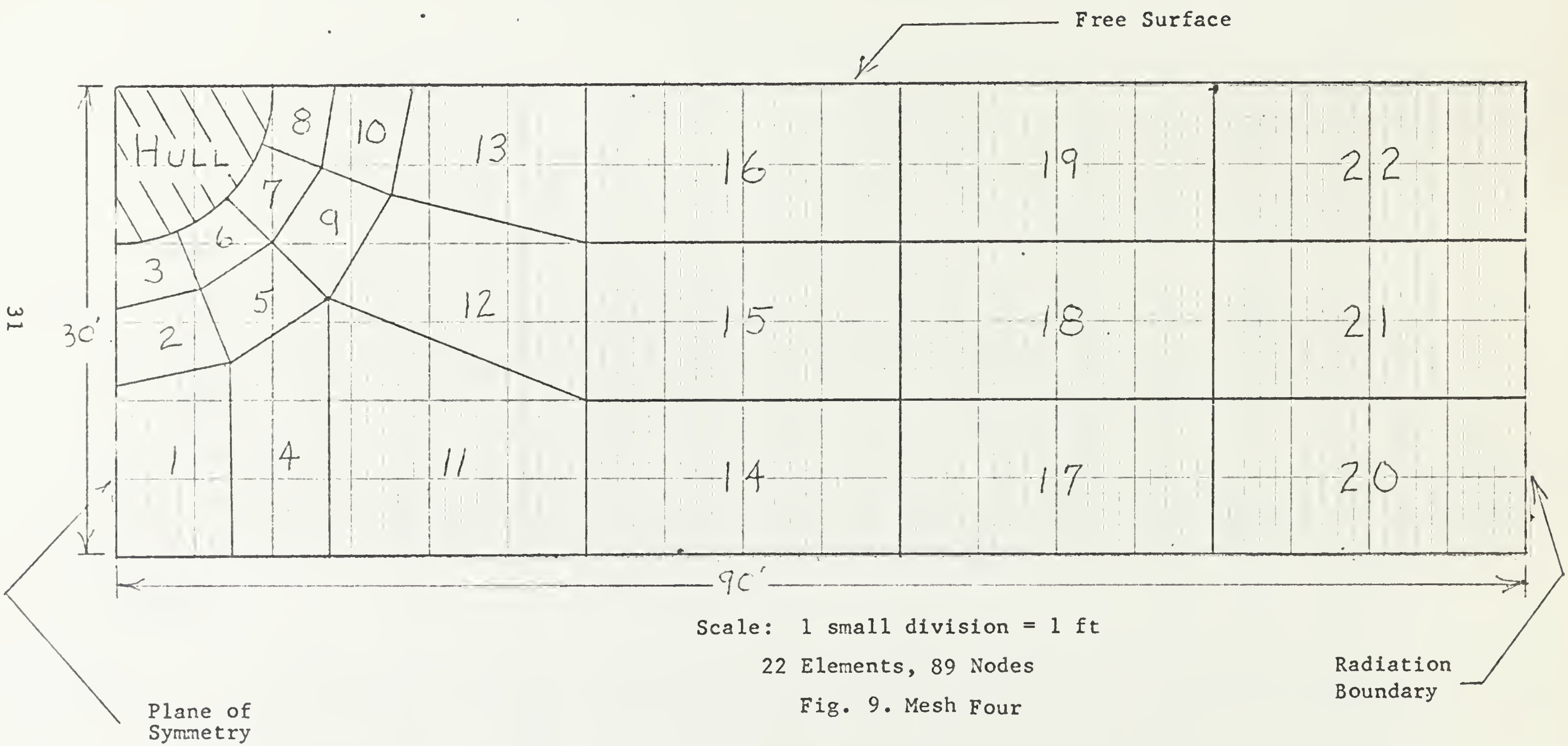
Fig. 7. Mesh Two



Scale: 1 small division = 1 ft

19 Elements, 78 Nodes

Fig. 8. Mesh Three



Scale: 1 small division = 1 ft

22 Elements, 89 Nodes

Fig. 9. Mesh Four

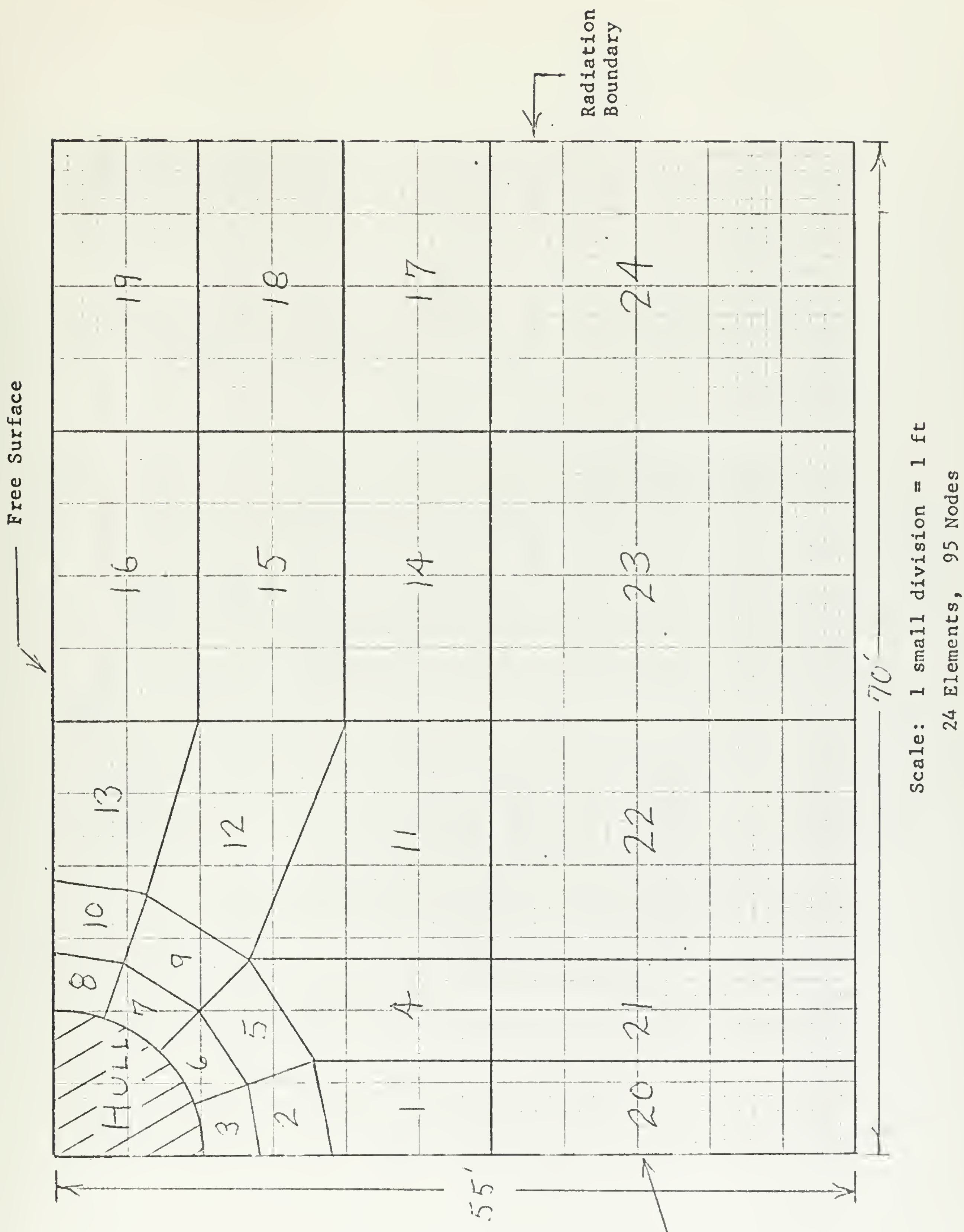


Fig. 10. Mesh Five

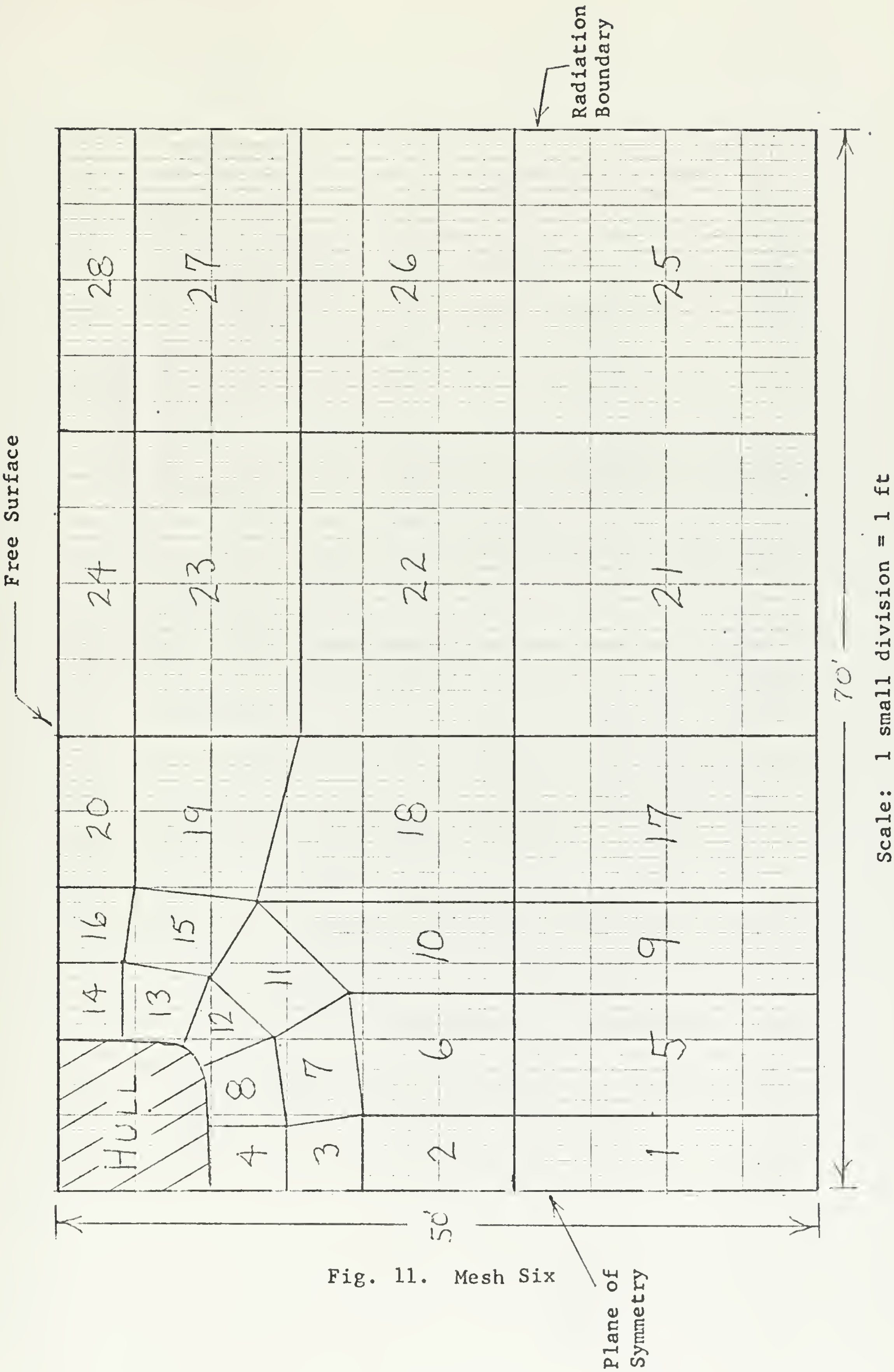
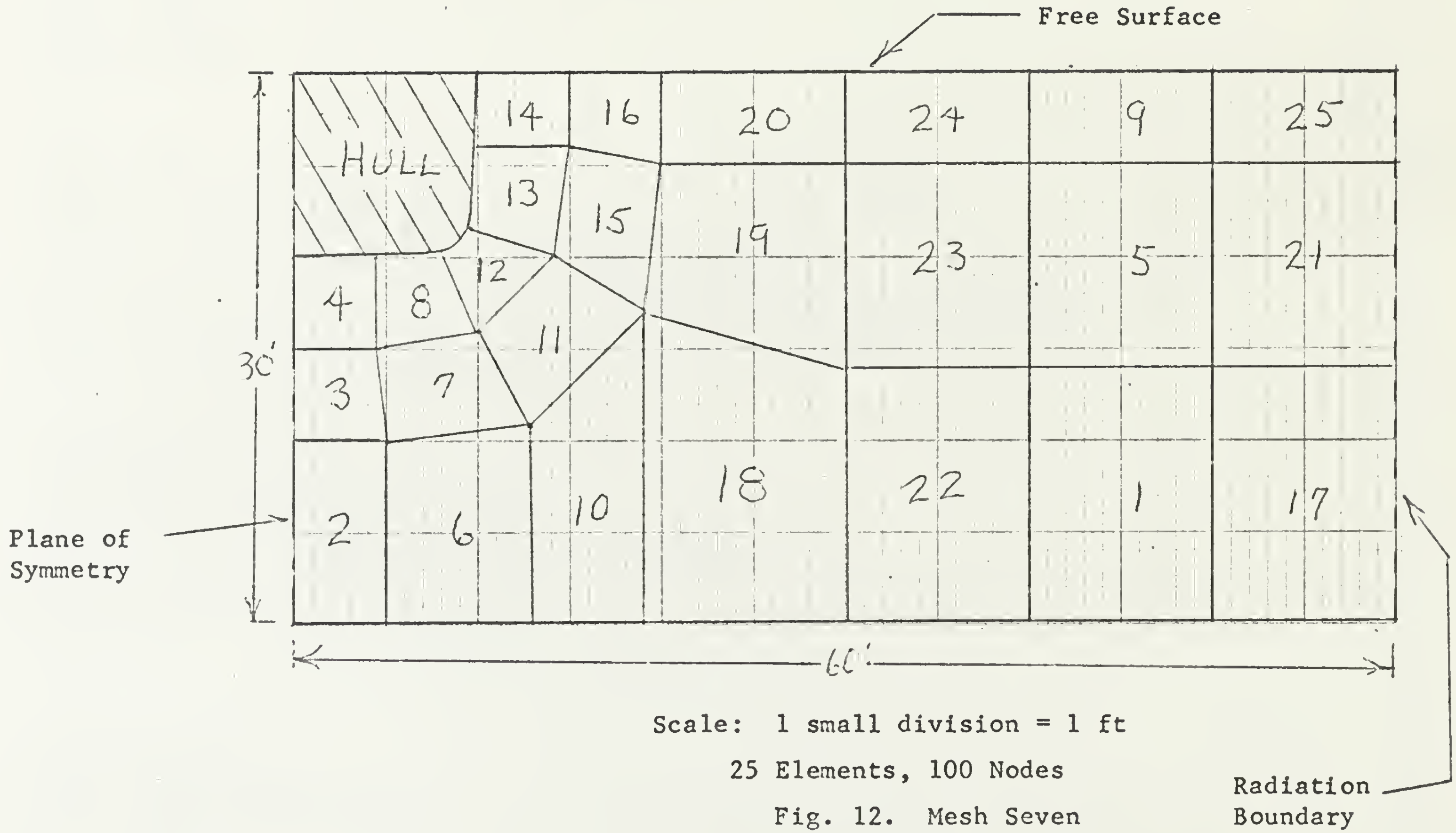


Fig. 11. Mesh Six



Scale: 1 small division = 1 ft

25 Elements, 100 Nodes

Fig. 12. Mesh Seven

COMPUTER PROGRAM

A list of the computer program, followed by definitions of symbols used, is given below.

```
      CALL INPUT
      CALL Q8STF
      CALL CLASS
      CALL INTFAC
      CALL SURWAV
      CALL RADAMP
      CALL ELIM1
    1 CALL ELIM2
      CALL ELIM3
      CALL BACSUB
      GO TO 1
      END

      SUBROUTINE INPUT
C     INPUT DATA FOR SHIP OSCILLATION WITH SURFACE WAVES
      IMPLICIT REAL*8(A-H,O-Y)
      IMPLICIT COMPLEX*16(Z)
      COMMON ZHRR(20,20),ZRRR(20),ZRR(40),ZP(200),
     1 X1(110),X2(110),H(110,110),DSU(20),QSU(20),V(20),R(110),RR(40),
     2 HR(40,40),QD(20),QU(20),DD(20),DU(20),OMEGA,RHO,G,WD,HB,
     6 DOMEGA,SM,NOMEGA,KOMEGA,
     3 NN(8,100),NSW(20),NRD(20),NSI(20),NT(200),NR(40),NWR(40),
     4 NNT,NET,NSWT,NRDT,NSIT,NTE,NRT
      DIMENSION HED(20)
C
      READ(5,500) HED
      WRITE(6,600) HED
      READ(5,501) NNT, NET,NNTN
      WRITE(6,601) NNT, NET, NNTN
      READ(5,503)(J,(NN(I,J), I=1,8),J=1,NET)
      WRITE(6,603)(J,(NN(I,J),I=1,8), J=1,NET)
      WRITE (6,613)
      DO 5 I=1,NNTN
    5 READ(5,502) J, X1(J), X2(J)
C
C     CALCULATE X AND Y COORDINATES WHERE NECESSARY
C
      DO 50 J=1,NET
      DO 50 I=2,8,2
      K = NN(I,J)
      IF(K) 40,50,50
   40 IF(I.LT.8) GO TO 45
      KP = NN(1,J)
      GO TO 48
   45 KP = NN(I+1,J)
   48 KM = NN(I-1,J)
      K = -K
```

```
      X1(K) = (X1(KP)+X1(KM))/2.0D0
      X2(K) = (X2(KP)+X2(KM))/2.0D0
      NN(I,J) = K
   50 CONTINUE
      WRITE(6,602) (I, X1(I), X2(I), I=1,NNT)
      READ(5,501) NSWT, NRDT, NSIT
      READ(5,505) (I, NSW(I), I=1,NSWT)
      READ(5,505) (I, NRD(I), I=1,NRDT)
      READ(5,505) (I,NSI(I), I=1,NSIT)
      WRITE(6,604) NSWT
      WRITE(6,605) (NSW(I), I=1,NSWT)
      WRITE(6,606) NRDT
      WRITE(6,605) (NRD(I), I=1,NRDT)
      WRITE(6,607) NSIT
      WRITE(6,605) (NSI(I), I=1,NSIT)
      READ(5,506) RHO, G, XCL
      WRITE(6,608) RHO, G, XCL
C
C  CALCULATE PARAMETERS FROM INPUT DATA
C
      XMAX = X1(1)
      YMAX = X2(1)
      YMIN = X2(1)
      DO 10 I = 2,NNT
      IF(X1(I).GT.XMAX) XMAX = X1(I)
      IF(X2(I).GT.YMAX) YMAX = X2(I)
   10 IF(X2(I).LT.YMIN) YMIN = X2(I)
      XMAX = XMAX - XCL
      WD = YMAX - YMIN
      WRITE(6,609) XMAX, WD
      XMAX = XCL
      YMIN = YMAX
      DO 20 I=1,NSIT
      J = NSI(I)
      IF(X1(J).GT.XMAX) XMAX = X1(J)
   20 IF(X2(J).LT.YMIN) YMIN = X2(J)
      SD = YMAX - YMIN
      HB = XMAX - XCL
      WRITE(6,610) SD,HB
      NSITM = NSIT - 1
      AS = 0.0D0
      DO 30 I = 1,NSITM
      J = NSI(I)
      JP = NSI(I+1)
   30 AS = AS + (X1(JP)-X1(J))*(YMAX-0.5D0*(X2(J)+X2(JP)))
      ADEL = 0.0D0
      NSITM = NSIT - 2
      DO 32 I=1,NSITM,2
```

```
      J = NSI(I)
      JP = NSI(I+1)
      JPP = NSI(I+2)
      XM1 = (X1(JP) + X1(J))/2.0D0
      XM2 = (X1(JP) + X1(JPP))/2.0D0
      YM1 = (X2(JP) + X2(J))/2.0D0
      YM2 = (X2(JP) + X2(JPP))/2.0D0
      XC1 = .375D0*X1(J) + .75D0*X1(JP) -.125D0*X1(JPP)
      YC1 = .375D0*X2(J) + .75D0*X2(JP) -.125D0*X2(JPP)
      XC2 = -.125D0*X1(J) + .75D0*X1(JP) + .375D0*X1(JPP)
      YC2 = -.125D0*X2(J) + .75D0*X2(JP) + .375D0*X2(JPP)
      ADEL = ADEL + (2.0D0/3.0D0)*((XM1-XC1)*(X2(J)-X2(JP))+ (YM1-YC1)*(
     #X1(JP)-X1(J)) + (XM2-XC2)*(X2(JP)-X2(JPP)) + (YM2-YC2)*(X1(JPP)-X1
     #(JP)))
   32 CONTINUE
      AS = AS + ADEL
      SM = RHO*AS
      SK = RHO*G*HB
      WRITE(6,611) SM, SK
      KOMEGA = 1
      READ(5,502) NOMEGA,OMEGA,DOMEGA
      NOMEG = NOMEGA + 1
      WRITE (6,612) NOMEG, OMEGA, DOMEGA
      RETURN
  500 FORMAT(20A4)
  501 FORMAT(3I5)
  502 FORMAT(I5,2F10.0)
  503 FORMAT(9I5)
  505 FORMAT(16I5)
  506 FORMAT(3F10.0)
  600 FORMAT(1H1,20A4)
  601 FORMAT(1H0, I5, 7H NODES, 5X,I5, 9H ELEMENTS,
     1 5X,I5,' SPECIFIED NODES',/)
  602 FORMAT(I5,2F10.3,I10,2F10.3,I10,2F10.3,I10,2F10.3,/)
  603 FORMAT('0ELEMENT  NODES',//' EL. #',16X,'NODE NO.',
     1 20X,'EL. #',16X,'NODE NO.',/
     29X,'1',4X,'2',4X,'3',4X,'4',4X,'5',4X,'6',4X,'7',4X,'8',
     3 14X,'1',4X,'2',4X,'3',4X,'4',4X,'5',4X,'6',4X,'7',4X,'8',//
     3 (9I5,I10,8I5,/))
  604 FORMAT(20H0BOUNDARY CONDITIONS,//I5,19H SURFACE WAVE NODES,/
     110H NODE NOS.)
  605 FORMAT(20I5)
  606 FORMAT( 1H0,I5,24H RADIATION DAMPING NODES,/10H NODE NOS.)
  607 FORMAT(1H0,I5,26H STRUCTURE INTERFACE NODES,/10H NODE NOS.)
  608 FORMAT(16H0WATER DENSITY =,F6.3,14H SLUGS/FT.**3,5X,4HG = ,F7.2,
     112H FT./SEC.**2,//13H X AT C.L. = ,F8.3, 4H FT.)
  609 FORMAT(20H0REGION SEMI-WIDTH =, F9.3,4H FT.,5X,13HWATER DEPTH =,
     1F8.3, 4H FT.)
```

```
610 FORMAT(13HOSHIP DRAFT =,F7.3,4H FT.,5X,11HSEMI-BEAM =,F7.3,4H FT.)
611 FORMAT(17HOSHIP SEMI-MASS =,F10.3,10H SLUGS/FT.,5X,20HVERTICAL STI
   1FFNESS =,1PE14.5,11H LB./FT.**2)
612 FORMAT(1H0, I3,16H VALUES OF OMEGA,/17H STARTING VALUE =,F7.3,
   111H RAD./SEC.,,4X,11HINCREMENT =,F6.3)
613 FORMAT(24HONODAL COORDINATES (FT.)//5H NO. ,4X,1HX,9X,1HY,10X,
   14H NO.,5X,1HX,9X,1HY,10X,
   24H NO.,5X,1HX,9X,1HY,10X,
   34H NO.,5X,1HX,9X,1HY,//)
    END

    SUBROUTINE Q8STF
C
C   SUBROUTINE Q8STF CALCULATES STIFFNESS MATRIX H FOR
C   AN EIGHT NODED ISO-PARAMETRIC ELEMENT USING FOUR
C   POINT GAUSS QUADRATURE
C
    IMPLICIT REAL*8(A-H,O-Y)
    IMPLICIT COMPLEX*16(Z)
    COMMON ZHRR(20,20),ZRRR(20),ZRR(40),ZP(200),
   1 X1(110),X2(110),H(110,110),DSU(20),QSU(20),V(20),R(110),RR(40),
   2 HR(40,40),QD(20),QU(20),DD(20),DU(20),OMEGA,RHO,G,WD,HB,
   6 DOMEGA,SM,NOMEGA,KOMEGA,
   3 NN(8,100),NSW(20),NRD(20),NSI(20),NT(200),NR(40),NWR(40),
   4 NNT,NET,NSWT,NRDT,NSIT,NTE,NRT
    DIMENSION GP(4),GW(4),SE(8),DX(8),DE(8),AJ(2,2),TH1(8),TH2(8),
   1 X(8),Y(8), HE(8,8)
    DATA GP/0.861136311594053, 0.339981043584856,
   1      -0.339981043584856, -0.861136311594053/,
   2  GW/0.347854845137454, 0.652145154862546,
   3      0.652145154862546, 0.347854845137454/
C
C   ZERO H AND HE MATRICES
C
    DO 1 I=1,NNT
    DO 1 J=1,NNT
  1 H(I,J) = 0.0D0
    DO 490 L=1,NET
    DO 10 I=1,8
    J=NN(I,L)
    X(I)= X1(J)
 10 Y(I)= X2(J)
C
C   CALCULATE HE MATRIX FOR EACH ELEMENT
C
    DO 20 J=1,8
    DO 20 I=1,8
```

```
20 HE(I,J) = 0.0D0
   DO 450 M=1,4
   DO 450 N=1,4
   XJ = GP(M)
   ET = GP(N)
   SE(1) = 0.25D0*(1.0D0-XJ)*(1.0D0-ET)*(-XJ-ET-1.0D0)
   SE(2) = 0.5D0*(1.0D0-XJ**2)*(1.0D0-ET)
   SE(3) = 0.25D0*(1.0D0+XJ)*(1.0D0-ET)*( XJ-ET-1.0D0)
   SE(4) = 0.5D0*(1.0D0+XJ)*(1.0D0-ET**2)
   SE(5) = 0.25D0*(1.0D0+XJ)*(1.0D0+ET)*( XJ+ET-1.0D0)
   SE(6) = 0.5D0*(1.0D0-XJ**2)*(1.0D0+ET)
   SE(7) = 0.25D0*(1.0D0-XJ)*(1.0D0+ET)*(-XJ+ET-1.0D0)
   SE(8) = 0.5D0*(1.0D0-XJ)*(1.0D0-ET**2)
   DX(1) = 0.25D0*(1.0D0-ET)*(2.0D0*XJ+ET)
   DX(2) = -XJ*(1.0D0-ET)
   DX(3) = 0.25D0*(1.0D0-ET)*(2.0D0*XJ-ET)
   DX(4) = 0.5D0*(1.0D0-ET**2)
   DX(5) = 0.25D0*(1.0D0+ET)*(2.0D0*XJ+ET)
   DX(6) = -XJ*(1.0D0+ET)
   DX(7) = 0.25D0*(1.0D0+ET)*(2.0D0*XJ-ET)
   DX(8) =-0.5D0*(1.0D0-ET**2)
   DE(2) = -0.5D0*(1.0D0-XJ**2)
   DE(1) = 0.25D0*(1.0D0-XJ)*(2.0D0*ET+XJ)
   DE(4) = -ET*(1.0D0+XJ)
   DE(3) = 0.25D0*(1.0D0+XJ)*(2.0D0*ET-XJ)
   DE(5) = 0.25D0*(1.0D0+XJ)*(2.0D0*ET+XJ)
   DE(6) =  0.5D0*(1.0D0-XJ**2)
   DE(7) = 0.25D0*(1.0D0-XJ)*(2.0D0*ET-XJ)
   DE(8) = -ET*(1.0D0-XJ)
   DO 30 I=1,2
   DO 30 J=1,2
30 AJ(I,J) = 0.0D0
   DO 40 I=1,8
   AJ(1,1) = AJ(1,1) + DX(I)*X(I)
   AJ(1,2) = AJ(1,2) + DX(I)*Y(I)
   AJ(2,1) = AJ(2,1) + DE(I)*X(I)
40 AJ(2,2) = AJ(2,2) + DE(I)*Y(I)
   DET = AJ(1,1)*AJ(2,2) - AJ(1,2)*AJ(2,1)
   TEMP = AJ(1,1)/DET
   AJ(1,1) = AJ(2,2)/DET
   AJ(2,2) = TEMP
   AJ(2,1) = -AJ(2,1)/DET
   AJ(1,2) = -AJ(1,2)/DET
   DO 50 J=1,8
    TH1(J)= AJ(1,1)*DX(J) + AJ(1,2)*DE(J)
50  TH2(J)= AJ(2,1)*DX(J) + AJ(2,2)*DE(J)
   TEMP = DET*GW(M)*GW(N)
   DO 100 J =1,8
```

```
      DO 100 K =J,8
  100 HE(J,K)=HE(J,K)+TEMP*(TH1(J)*TH1(K)+TH2(J)*TH2(K))
  450 CONTINUE
      DO 460 I=1,7
      JL = I + 1
      DO 460 J = JL,8
  460 HE(J,I) = HE(I,J)
C
C    ADD TO STIFFNESS MATRIX
C
      DO 470 I=1,8
      DO 470 J=1,8
      II = NN(I,L)
      JJ = NN(J,L)
  470 H(II,JJ) = H(II,JJ) + HE(I,J)
  490 CONTINUE
      RETURN
      END


      SUBROUTINE INTFAC
C
C     SUBROUTINE INTFAC CALCULATES COUPLING VECTOR
C     COMPONENTS AT THE SHIP INTERFACE FOR AN EIGHT NODED
C     ELEMENT USING THREE POINT GAUSS QUADRATURE
C
      IMPLICIT REAL*8(A-H,O-Y)
      IMPLICIT COMPLEX*16(Z)
      COMMON ZHRR(20,20),ZRRR(20),ZRR(40),ZP(200),
     1 X1(110),X2(110),H(110,110),DSU(20),QSU(20),V(20),R(110),RR(40),
     2 HR(40,40),QD(20),QU(20),DD(20),DU(20),OMEGA,RHO,G,WD,HB,
     6 DOMEGA,SM,NOMEGA,KOMEGA,
     3 NN(8,100),NSW(20),NRD(20),NSI(20),NT(200),NR(40),NWR(40),
     4 NNT,NET,NSWT,NRDT,NSIT,NTE,NRT
      DIMENSION SE(3),DX(3), GP(3), GW(3)
      DATA GP/0.774596669241483,0.0D0,-0.774596669241483/,
     1 GW/0.555555555555556,0.888888888888889,0.555555555555556/
C
      DO 1 I=1,NSIT
    1 V(I) = 0.0D0
      IM = NSIT - 1
      DO 100 I=1,IM,2
      J = NSI(I)
      JP = NSI(I + 1)
      JPP = NSI(I+2)
      DO 50 N=1,3
      XJ = GP(N)
      SE(1) = 0.5D0*(1.0D0-XJ)*(-XJ)
```

```
      SE(2) = (1.0D0-XJ**2)
      SE(3) = 0.5D0*(1.0D0+XJ)*(XJ)
      DX(1) = (XJ-0.5D0)
      DX(2) = -2.0D0*XJ
      DX(3) = (XJ+0.5D0)
      SUM = DX(1)*X1(J) +DX(2)*X1(JP) +DX(3)*X1(JPP)
      V(I) =V(I) + SUM*SE(1)*GW(N)
      V(I+1) = V(I+1) + SUM*SE(2)*GW(N)
      V(I+2) = V(I+2) +SUM*SE(3)*GW(N)
   50 CONTINUE
  100 CONTINUE
      RETURN
      END


      SUBROUTINE SURWAV
C
C     SUBROUTINE SURWAV CALCULATES ELEMENTS OF SURFACE WAVE
C     MATRIX FOR EIGHT NODED ELEMENTS USING THREE POINT
C     GAUSS QUADRATURE. RESULTS ARE STORED IN THREE VECTORS
C     QD(I) FOR DIAGONAL, QU(I) FOR ABOVE DIAGONAL, AND
C     QSU(I) FOR TWO ROWS ABOVE DIAGONAL
C
      IMPLICIT REAL*8(A-H,O-Y)
      IMPLICIT COMPLEX*16(Z)
      COMMON ZHRR(20,20),ZRRR(20),ZRR(40),ZP(200),
     1 X1(110),X2(110),H(110,110),DSU(20),QSU(20),V(20),R(110),RR(40),
     2 HR(40,40),QD(20),QU(20),DD(20),DU(20),OMEGA,RHO,G,WD,HB,
     6 DOMEGA,SM,NOMEGA,KOMEGA,
     3 NN(8,100),NSW(20),NRD(20),NSI(20),NT(200),NR(40),NWR(40),
     4 NNT,NET,NSWT,NRDT,NSIT,NTE,NRT
      DIMENSION SE(3), GP(3), GW(3)
      DATA GP/0.774596669241483,0.0D0,-0.774596669241483/,
     1 GW/0.555555555555556,0.888888888888889,0.555555555555556/
C
      DO 1 I=1,NSWT
      QD(I) = 0.0D0
      QU(I) = 0.0D0
    1 QSU(I) = 0.0D0
      IM = NSWT-1
      DO 75 I=1,IM,2
      M = NSW(I)
      MP = NSW(I+2)
      A = DABS(X1(MP)-X1(M))
      DO 50 N=1,3
      XJ = GP(N)
      SE(1) = 0.5D0*(1.0D0+XJ)*(XJ)
      SE(2) = (1.0D0-XJ**2)
```

```
      SE(3) = 0.5D0*(1.0D0-XJ)*(-XJ)
      COEF = (A/(2.0D0*G))*GW(N)
      QD(I) = QD(I) + COEF*SE(1)*SE(1)
      QD(I+1) = QD(I+1) + COEF*SE(2)*SE(2)
      QD(I+2) = QD(I+2) + COEF*SE(3)*SE(3)
      QU(I) = QU(I) + COEF*SE(1)*SE(2)
      QU(I+1) = QU(I+1) + COEF*SE(2)*SE(3)
      QSU(I) = QSU(I) + COEF*SE(1)*SE(3)
      QSU(I+1) = QSU(I+1) + 0.0D0
   50 CONTINUE
   75 CONTINUE
      RETURN
      END


      SUBROUTINE RADAMP
C
C     SUBROUTINE RADAMP CALCULATES ELEMENTS OF RADIATION
C     DAMPING MATRIX FOR EIGHT NODED ELEMENTS USING THREE
C     POINT GAUSS QUADRATURE. RESULTS ARE STORED IN THREE
C     VECTORS    DD(I) FOR DIAGONAL, DU(I) FOR ABOVE
C     DIAGONAL, AND DSU(I) FOR TWO ROWS ABOVE THE DIAGONAL
C
      IMPLICIT REAL*8(A-H,O-Y)
      IMPLICIT COMPLEX*16(Z)
      COMMON ZHRR(20,20),ZRRR(20),ZRR(40),ZP(200),
     1 X1(110),X2(110),H(110,110),DSU(20),QSU(20),V(20),R(110),RR(40),
     2 HR(40,40),QD(20),QU(20),DD(20),DU(20),OMEGA,RHO,G,WD,HB,
     6 DOMEGA,SM,NOMEGA,KOMEGA,
     3 NN(8,100),NSW(20),NRD(20),NSI(20),NT(200),NR(40),NWR(40),
     4 NNT,NET,NSWT,NRDT,NSIT,NTE,NRT
      DIMENSION SE(3), GP(3), GW(3)
      DATA GP/0.774596669241483,0.0D0,-0.774596669241483/,
     1 GW/0.555555555555556,0.888888888888889,0.555555555555556/
C
      DO 1 I=1,NRDT
      DD(I) = 0.0D0
      DU(I) = 0.0D0
    1 DSU(I) = 0.0D0
      IM = NRDT-1
      DO 75 I=1,IM,2
      M = NRD(I)
      MP = NRD(I+2)
      A = DABS(X2(MP)-X2(M))
      DO 50 N=1,3
      ET = GP(N)
      SE(1) = 0.5D0*(1.0D0-ET)*(-ET)
      SE(2) =  (1.0D0-ET**2)
```

```
      SE(3) = 0.5D0*(1.0D0+ET)*(ET)
      COEF = (A/2.0D0)*GW(N)
      DD(I) = DD(I) + COEF*SE(1)*SE(1)
      DD(I+1) = DD(I+1) + COEF*SE(2)*SE(2)
      DD(I+2) = DD(I+2) + COEF*SE(3)*SE(3)
      DU(I) = DU(I) + COEF*SE(1)*SE(2)
      DU(I+1) = DU(I+1) + COEF*SE(2)*SE(3)
      DSU(I) = DSU(I) + COEF*SE(1)*SE(3)
      DSU(I+1) = DSU(I+1) + 0.0D0
   50 CONTINUE
   75 CONTINUE
      RETURN
      END


      SUBROUTINE WAVEL(OMEGA,G,WD,C)
C
C     SUBROUTINE WAVEL CALCULATES WAVE VELOCITY FOR DEPTH WD
C
      IMPLICIT REAL*8(A-H,O-Z)
      C0 = G/OMEGA
      C1 = C0
      P = OMEGA*WD
    1 C2 = C0*DTANH(P/C1)
      CRIT = DABS(C1 - C2)/C2
      C1 = (C2 + C1)*0.5D0
      IF(CRIT.GT.1.0D-6) GO TO 1
      C = C1
      RETURN
      END


      SUBROUTINE CLASS
C
C     SUBROUTINE CLASS CLASSIFIES NODE TYPES AS FOLLOWS
C     #1 FOR GENERAL NODES, #2 FOR SHIP INTERFACE NODES
C     #3 FOR SURFACE WAVE NODES, #4 FOR RADIATION NODES
C
      IMPLICIT REAL*8(A-H,O-Y)
      IMPLICIT COMPLEX*16(Z)
      COMMON ZHRR(20,20),ZRRR(20),ZRR(40),ZP(200),
     1 X1(110),X2(110),H(110,110),DSU(20),QSU(20),V(20),R(110),RR(40),
     2 HR(40,40),QD(20),QU(20),DD(20),DU(20),OMEGA,RHO,G,WD,HB,
     6 DOMEGA,SM,NOMEGA,KOMEGA,
     3 NN(8,100),NSW(20),NRD(20),NSI(20),NT(200),NR(40),NWR(40),
     4 NNT,NET,NSWT,NRDT,NSIT,NTE,NRT
C
      DO 1 I=1,NNT
```

```
    1 NT(I) = 1
      DO 2 I=1,NSIT
      J = NSI(I)
    2 NT(J) = 2
      DO 3 I=1,NSWT
      J = NSW(I)
    3 NT(J) = 3
      DO 4 I=1,NRDT
      J = NRD(I)
    4 NT(J) = 4
      WRITE (6,110)
      WRITE(6,100) (I, NT(I), I=1,NNT)
  100 FORMAT(2I5,10X,2I5,10X,2I5,10X,2I5,/)
  110 FORMAT('ONODE  TYPE',9X,'NODE  TYPE',9X,'NODE  TYPE',
     1 9X,'NODE  TYPE',//)
      RETURN
      END


      SUBROUTINE ELIM1
C
C      SUBROUTINE ELIM1 ELIMINATES FIRST TWO PARTITIONS OF PRESSURE
C      VECTOR FROM H(I,J)
C
      IMPLICIT REAL*8(A-H,O-Y)
      IMPLICIT COMPLEX*16(Z)
      COMMON ZHRR(20,20),ZRRR(20),ZRR(40),ZP(200),
     1 X1(110),X2(110),H(110,110),DSU(20),QSU(20),V(20),R(110),RR(40),
     2 HR(40,40),QD(20),QU(20),DD(20),DU(20),OMEGA,RHO,G,WD,HB,
     6 DOMEGA,SM,NOMEGA,KOMEGA,
     3 NN(8,100),NSW(20),NRD(20),NSI(20),NT(200),NR(40),NWR(40),
     4 NNT,NET,NSWT,NRDT,NSIT,NTE,NRT
C
      DO 100 I=1,NNT
  100 R(I) = 0.0D0
      DO 101 I=1,NSIT
      J = NSI(I)
  101 R(J) = V(I)
      NNTP = NNT + 1
      NNTM = NNT - 1
      DO 400 I=1,NNTM
      IF(NT(I).GE.3) GO TO 400
      IF(H(I,I).EQ.0.0D0) GO TO 501
C
C      FIND BAND LIMITS LL, LU
C
      DO 1 J=1,I
      IF(H(I,J).NE.0.0D0) GO TO 2
```

```
    1 CONTINUE
    2 LL = J
      JM = NNTP - I
      DO 3 J=1,JM
      K = NNTP - J
      IF(H(I,K).NE.0.0D0) GO TO 4
    3 CONTINUE
    4 LU = K
      IM = I-1
      IP = I+1
      DO 10 K=1,IM
      IF(NT(K).LE.2) GO TO 10
      IF(H(K,I).EQ.0.0D0) GO TO 10
      TEMP = H(K,I)/H(I,I)
      DO 5 J = LL,LU
    5 H(K,J) = H(K,J)-TEMP*H(I,J)
      IF(R(I).NE.0.0D0) R(K) =R(K)-TEMP*R(I)
   10 CONTINUE
      DO 20 K = IP,NNT
      IF(H(K,I).EQ.0.0D0) GO TO 20
      TEMP = H(K,I)/H(I,I)
      DO 11 J = LL,LU
   11 H(K,J) = H(K,J) - TEMP*H(I,J)
      IF(R(I).NE.0.0D0) R(K) = R(K) - TEMP*R(I)
   20 CONTINUE
  400 CONTINUE
C
C     REARRANGE REMAINING COMPONENTS OF PRESSURE VECTOR FOR USE IN ELIM2
C
      IE = 0
      DO 410 I=1,NSWT
      J = NSW(I)
      IF(NT(J).NE.3) GO TO 405
      K = I-IE
      NR(K) = J
      GO TO 410
  405 IE = IE+1
  410 CONTINUE
      DO 412 I=1,NRDT
      J = K+I
  412 NR(J) = NRD(I)
      NRT = K+NRDT
      DO 420 I=1,NSWT
      K = NSW(I)
      DO 413 J = I,NRT
      IF(NR(J).EQ.K) GO TO 420
  413 CONTINUE
  420 NWR(I) = J
```

```
      RETURN
  501 WRITE(6,600) I,H(I,I)
  600 FORMAT(11H ELIM1 I = , I3, 4X, 9HH(I,I) = , E12.3)
      STOP
      END


      SUBROUTINE ELIM2
C
C     SUBROUTINE ELIM2 ELIMINATES THIRD PARTITION OF PRESSURE VECTOR
C    FOR A SPECIFIC VALUE OF OMEGA**2 AND ADDS CONTRIBUTION
C     OF QO MATRIX TO H MATRIX
C
      IMPLICIT REAL*8(A-H,O-Y)
      IMPLICIT COMPLEX*16(Z)
      COMMON ZHRR(20,20),ZRRR(20),ZRR(40),ZP(200),
     1 X1(110),X2(110),H(110,110),DSU(20),QSU(20),V(20),R(110),RR(40),
     2 HR(40,40),QD(20),QU(20),DD(20),DU(20),OMEGA,RHO,G,WD,HB,
     6 DOMEGA,SM,NOMEGA,KOMEGA,
     3 NN(8,100),NSW(20),NRD(20),NSI(20),NT(200),NR(40),NWR(40),
     4 NNT,NET,NSWT,NRDT,NSIT,NTE,NRT
C
C     TRANSFER FROM H(I,J) TO HR(I,J) AND FROM R(I) TO RR(I)
C
      DO 1 I=1,NRT
      RR(I) = 0.0D0
      DO 1 J=1,NRT
    1 HR(I,J) = 0.0D0
      TEMP = RHO*(OMEGA**2)
      DO 5 I=1,NRT
      II = NR(I)
      RR(I) = TEMP*R(II)
      DO 5 J=1,NRT
      JJ = NR(J)
    5 HR(I,J) = H(II,JJ)
C
C     ADD CONTRIBUTION - QO * OMEGA**2
C
      TEMP = OMEGA**2
      DO10 I =1,NSWT
      II = NWR(I)
   10 HR(II,II) = HR(II,II) - TEMP*QD(I)
      NSWTM = NSWT-1
      DO 12 I=1,NSWTM
      II = NWR(I)
      JJ = NWR(I+1)
      QI = TEMP*QU(I)
      HR(II,JJ) = HR(II,JJ) - QI
```

```
   12 HR(JJ,II) = HR(JJ,II) - QI
      NSWTS = NSWT-2
      DO 25 I=1,NSWTS
      II = NWR(I)
      JJ = NWR(I+2)
      QS = TEMP*QSU(I)
      HR(II,JJ) = HR(II,JJ) - QS
   25 HR(JJ,II) = HR(JJ,II) - QS
C
C     ELIMINATE THIRD PARTITION OF PRESSURE VECTOR
C
      LU = NRT-NRDT
      DO 30 I=1,LU
      IF(HR(I,I).EQ.0.0D0) GO TO 501
      IP = I + 1
      DO 30 J=IP,NRT
      IF(HR(J,I).EQ.0.0D0) GO TO 30
      TEMP = HR(J,I)/HR(I,I)
      HR(J,I) = 0.0D0
      RR(J) = RR(J) - TEMP*RR(I)
      DO 30 K=IP,NRT
      IF(HR(I,K).EQ.0.0D0) GO TO 30
      HR(J,K) = HR(J,K) - TEMP*HR(I,K)
   30 CONTINUE
      RETURN
  501 WRITE(6,600) I,NR(I)
  600 FORMAT(28H HR(I,I) = 0. IN ELIM2, I = , I2,4X,11HNODE NO. = ,I3)
      STOP
      END


      SUBROUTINE ELIM3
C
C     SUBROUTINE ELIM3 MAKES HRR MATRIX COMPLEX AND ADDS
C     CONTRIBUTION OF RADIATION DAMPING TO HRR.
C     IT THEN CALLS SOLUTION ROUTINE
C
      IMPLICIT REAL*8(A-H,O-Y)
      IMPLICIT COMPLEX*16(Z)
      COMMON ZHRR(20,20),ZRRR(20),ZRR(40),ZP(200),
     1 X1(110),X2(110),H(110,110),DSU(20),QSU(20),V(20),R(110),RR(40),
     2 HR(40,40),QD(20),QU(20),DD(20),DU(20),OMEGA,RHO,G,WD,HB,
     6 DOMEGA,SM,NOMEGA,KOMEGA,
     3 NN(8,100),NSW(20),NRD(20),NSI(20),NT(200),NR(40),NWR(40),
     4 NNT,NET,NSWT,NRDT,NSIT,NTE,NRT
      DIMENSION RRR(20),HRR(20,20)
C
C      TRANSFER TO HRR(I,J) AND RRR(I)
```

```
C
      DO 1 I=1,NRDT
      RRR(I)=0.0D0
      DO 1 J=1,NRDT
    1 HRR(I,J)=0.0D0
      LU = NRT-NRDT
      DO 5 I=1,NRDT
      II = I+LU
      RRR(I) = RR(II)
      DO 5 J=1,NRDT
      JJ = J+LU
    5 HRR(I,J) = HR(II,JJ)
C
C     ADD (1/C)DAMP TO HRR(I,J) AND MAKE COMPLEX FOR SPECIFIC OMEGA
C
      CALL WAVEL(OMEGA,G,WD,C)
      DO 10 I=1,NRDT
      ZRRR(I) = DCMPLX(RRR(I),0.0D0)
      DO 10 J=1,NRDT
   10 ZHRR(I,J) = DCMPLX(HRR(I,J),0.0D0)
      NRDTM = NRDT-1
      DO 15 I=1,NRDTM
      IP = I+1
      ZHRR(I,IP) = ZHRR(I,IP) + DCMPLX(0.0D0,(OMEGA/C)*DU(I))
   15 ZHRR(IP,I) = ZHRR(IP,I) + DCMPLX(0.0D0,(OMEGA/C)*DU(I))
      DO 20 I=1,NRDT
   20 ZHRR(I,I) = ZHRR(I,I) + DCMPLX(0.0D0,(OMEGA/C)*DD(I))
      NRDTS = NRDT-2
      DO 25 I=1,NRDTS
      J = I+2
      ZHRR(I,J) = ZHRR(I,J) + DCMPLX(0.0D0,(OMEGA/C)*DSU(I))
   25 ZHRR(J,I) = ZHRR(J,I) + DCMPLX(0.0D0,(OMEGA/C)*DSU(I))
      CALL SOLUN
      WRITE (6,107) OMEGA,C
      X1(100) = C
  107 FORMAT('1 PRESSURES',10X,' OMEGA = ',F10.4,' RAD/SEC ',10X,
     3 ' C = ',F10.4,' FT/SEC',//
     1'0NODE',5X,'REAL PART',7X,'IMAGINARY PART',11X,
     2 ' ABSOLUTE VALUE    PHASE ANGLE',//)
      RETURN
      END


      SUBROUTINE SOLUN
C
C     SUBROUTINE SOLUN SOLVES NRDT SIMULTANEOUS EQUATIONS
C     WITH NRDT UNKNOWNS AND PLACES THE RESULT IN VECTOR P
C
```

```
      IMPLICIT REAL*8(A-H,O-Y)
      IMPLICIT COMPLEX*16(Z)
      COMPLEX*16 SUM(20,20), P(20)
      COMMON ZHRR(20,20),ZRRR(20),ZRR(40),ZP(200),
     1 X1(110),X2(110),H(110,110),DSU(20),QSU(20),V(20),R(110),RR(40),
     2 HR(40,40),QD(20),QU(20),DD(20),DU(20),OMEGA,RHO,G,WD,HB,
     6 DOMEGA,SM,NOMEGA,KOMEGA,
     3 NN(8,100),NSW(20),NRD(20),NSI(20),NT(200),NR(40),NWR(40),
     4 NNT,NET,NSWT,NRDT,NSIT,NTE,NRT
C
      EQUIVALENCE (ZHRR(1,1),SUM(1,1))
      EQUIVALENCE (ZRRR(1),P(1))
      DO 10 N=1,NRDT
      N1 = N+1
      P(N) = P(N)/SUM(N,N)
      IF(N-NRDT) 20,50,20
   20 DO 30 J=N1,NRDT
      IF(CDABS(SUM(N,J))) 25,30,25
   25 SUM(N,J) = SUM(N,J)/SUM(N,N)
      DO 35 I=J,NRDT
      SUM(I,J) = SUM(I,J)-SUM(I,N)*SUM(N,J)
   35 SUM(J,I) = SUM(I,J)
      P(J) = P(J)-SUM(J,N)*P(N)
   30 CONTINUE
   10 CONTINUE
C
C     BACKSUBSTITUTION  ROUTINE
C
   50 N1 = N
      N = N-1
      IF(N) 70,70,55
   55 DO 60  J=N1,NRDT
   60 P(N) = P(N)-SUM(N,J)*P(J)
      GO TO 50
   70 RETURN
      END


      SUBROUTINE BACSUB
C
C     SUBROUTINE BACSUB USES BACKSUBSTITUTION AND THE
C     RESULTS OF SOLUN TO FIND THE PRESSURE VECTOR,FINDING
C     P3 FIRST,THEN P(2&1). IT ALSO ARRANGES VALUES OF P
C     IN ORIGINAL ORDER
C
      IMPLICIT REAL*8(A-H,O-Y)
      IMPLICIT COMPLEX*16(Z)
      COMMON ZHRR(20,20),ZRRR(20),ZRR(40),ZP(200),
```

```
     1 X1(110),X2(110),H(110,110),DSU(20),QSU(20),V(20),R(110),RR(40),
     2 HR(40,40),QD(20),QU(20),DD(20),DU(20),OMEGA,RHO,G,WD,HB,
     6 DOMEGA,SM,NOMEGA,KOMEGA,
     3 NN(8,100),NSW(20),NRD(20),NSI(20),NT(200),NR(40),NWR(40),
     4 NNT,NET,NSWT,NRDT,NSIT,NTE,NRT
      DIMENSION PAB(20), DP(20)
      C = X1(100)
C
C     CHANGE HR(I,J) AND RR(I) TO COMPLEX
C
      LU=NRT-NRDT
      DO 10 I=1,LU
   10 ZRR(I) = DCMPLX(RR(I),0.0D0)
      II = 0
      DO 20 I=NSWT,NRT
      II = II+1
   20 ZRR(I) = ZRRR(II)
C
C     BACKSUBSTITUTION FOR P3
C
      N = NSWT
   30 N1 = N
      N = N-1
      IF(N) 60,60,40
   40 DO 50 J=N1,NRT
   50 ZRR(N) = ZRR(N)-HR(N,J)*ZRR(J)
      ZRR(N) = ZRR(N)/HR(N,N)
      GO TO 30
   60 CONTINUE
C
C     CHANGE R(I) AND H(I,J) TO COMPLEX AND TRANSFER ZRR(I)
C     TO ZP(I), ARRANGING IN ORIGINAL ORDER
C
      DO 70 I=1,NNT
   70 ZP(I) = DCMPLX(0.0D0,0.0D0)
      DO 75 I=1,NRT
      II = NR(I)
   75 ZP(II) = ZRR(I)
C
C     BACKSUBSTITUTION  FOR  P2 & P1
C
      N1 = NNT
   80 N = N1
      N1 = N-1
      IF(NT(N).GT.2) GO TO 80
      IF(N) 95,95,85
   85 ZTEMP = DCMPLX((OMEGA**2)*RHO*R(N),0.0D0)
      DO 90 J=1,NNT
```

```
   90 ZTEMP = ZTEMP-H(N,J)*ZP(J)
      ZP(N) = ZTEMP/H(N,N)
      GO TO 80
   95 CONTINUE
C
C     FIND ABSOLUTE VALUE AND PHASE ANGLES OF PRESSURES AND WRITE
C
      DO 105 I=1,NNT
      ABVAL = CDABS(ZP(I))
      A = ZP(I)
      B = ZP(I)*(0.0D0,-1.0D0)
      THETA = 57.29578*DATAN2(B,A)
  105 WRITE(6,110) I,ZP(I),ABVAL,THETA
C
C     FIND ADDED MASS AND DAMPING
C
      ZTEMP = (0.0D0,0.0D0)
      DO 200 I=1,NSIT
      J = NSI(I)
  200 ZTEMP = ZTEMP + V(I)*ZP(J)
      AM = ZTEMP
      AD = ZTEMP*(0.0D0,-1.0D0)
      CSM = SM*(OMEGA**2)
      AMAS = AM/CSM
      ADAMP = -AD/CSM
      WRITE (6,300) AMAS, ADAMP
C
C      ENERGY CHECK TO INSURE MESH CORRECTLY SOLVES PROBLEM
C
      EIN = -(AD*OMEGA)/2.0D0
      NRDTM = NRDT-1
      NRDTN = NRDT-2
      DO 210 I=1,NRDT
      J = NRD(I)
      PAB(I) = CDABS(ZP(J))
  210 DP(I) = PAB(I)*DD(I)
      DO 220 I=1,NRDTM
  220 DP(I) = DP(I) + PAB(I+1)*DU(I)
      DO 230 I=1,NRDTN
  230 DP(I) = DP(I) + PAB(I+2)*DSU(I)
      DO 240 I=2,NRDT
  240 DP(I) = DP(I) + PAB(I-1)*DU(I-1)
      DO 250 I=3,NSIT
  250 DP(I) = DP(I) + PAB(I-2)*DSU(I-2)
      EOUT = 0.0D0
      DO 260 I=1,NRDT
  260 EOUT = EOUT + DP(I)*PAB(I)
      EOUT = EOUT/(2.0D0*RHO*C)
```

```
      WRITE (6,310) EIN, EOUT
C
C      CALCULATE HEAVE FORCE AND WAVE AMPLITUDE
C
      DELTA = ((OMEGA**2)*HB)/G
      I = NRD(NRDT)
      TEMP = CDABS(ZP(I))
      ABAR = TEMP/(RHO*G)
      FB = RHO*G*HB
      FAM = (1.0D0+AMAS)*(OMEGA**2)*SM
      F1 = FB - FAM
      F2 = ADAMP*(OMEGA**2)*SM
      FT = ((F1**2) + (F2**2))**0.5D0
      FBAR = FT/FB
      PHI = 57.29578D0*DATAN2(F2,F1)
      WRITE (6,320) DELTA, FBAR, ABAR, PHI
C
C     CHANGE VALUE OF OMEGA AND SOLVE PROBLEM AGAIN
C     STARTING AT ELIM2
C
      IF(KOMEGA.GT.NOMEGA) GO TO 500
      OMEGA = OMEGA + DOMEGA
      KOMEGA = KOMEGA + 1
      RETURN
  110 FORMAT(I5,1P2E16.5,10X,1PE16.5,0PF12.4,//)
  300 FORMAT('0ADDED MASS = ',F10.5,10X,'DAMPING COEF = ',F10.5,//)
  310 FORMAT('0ENERGY CHECK',/'0AVERAGE POWER IN = ',1PE16.5,10X,
     1 ' AVERAGE POWER OUT = ',1PE16.5,//)
  320 FORMAT('0DELTA = ',F10.4,10X,' HEAVE FORCE = ',F10.4,10X,//
     # ' WAVE AMPLITUDE = ',F10.4,10X,' PHASE ANGLE = ',F10.4,//)
  500 STOP
      END
```

NOTATION USED IN COMPUTER PROGRAM

ABAR	Nondimensional wave amplitude
ADAMP	Nondimensional damping coefficient
ADEL	Area correction for chord areas
AJ(I,J)	Jacobian matrix (and its inverse)
AMAS	Nondimensional added mass
AS	Hull area
C	Wave velocity (calculated by WAVEL)
DD(I)	Diagonal element of D at damping Node I
DE(I)	Derivative of shape function w.r.t. η
DELTA	Nondimensional frequency
DOMEGA	Increment in omega
DSU(I)	Super above diagonal element of D for damping Nodes I, I+2
DU(I)	Above diagonal element of D for damping Nodes I, I+1
DX(I)	Derivative of shape function w.r.t. ξ
ET	Variable η
FBAR	Nondimensional heave force
G	Gravitational acceleration
GP(I)	Abscissa for Gauss quadrature
GW(I)	Weight factor for Gauss quadrature
H(I,J)	Assembled stiffness matrix
HB	Ship half-beam
HE(I,J)	Element stiffness matrix
HED(I)	Title (80 characters in 20 words)
HR(I,J)	Reduced stiffness matrix in ELIM2

HRR(I,J)	Reduced stiffness matrix in ELIM3
KOMEGA	Counter for omega (set to 1 in INPUT)
NET	Total number of elements
NN(I,J)	Node No. of Node I, element J
NNT	Total number of nodes
NNTN	Total number of nodes where x and y coordinates are specified
NOMEGA	Number of values of Omega
NR(I)	Node No. for row I of HR(I,J)
NRD(I)	Node no. of radiation damping Node I
NRDT	Total number of radiation damping nodes
NRT	Dimension of HR
NSI(I)	Node no. of ship interface node I
NSIT	Total number of ship interface nodes
NSW(I)	Node no. of surface wave node I
NSWT	Total number of surface wave nodes
NT(I)	Node type of node I
NWR(I)	Row number in HR of I'th surface wave node
OMEGA	Circular frequency, ω
PHI	Force phase angle relative to hull motion
QD(I)	Diagonal element of Q_o at surface node I
QSU(I)	Super above diagonal element of Q_o for surface nodes I, I+2
QU(I)	Above diagonal element of Q_o for surface nodes I, I+1
R(I)	Right hand side for ELIM1
RR(I)	Right hand side for ELIM2
RRR(I)	Right hand side for ELIM3
RHO	Water density .
SD	Ship's draft

SE(I)	Shape function for element node I
SK	Vertical stiffness/unit length (half ship)
SM	Mass/unit length (half-ship)
V(I)	Coupling vector at interface node I
WD	Water depth
X(I)	Element nodal coordinate (X1)
X1(I)	x coordinate of node I
X2(I)	y coordinate of node I
XJ	Variable ξ
XMAX	Region half width
Y(I)	Element nodal coordinate (X2)
Z	Used as prefix to denote complex form of a variable. Example: ZRRR(I) is complex right-hand side in ELIM3.
XCL	x coordinate of ship's centerline

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13. ABSTRACT

Forced harmonic heaving motion of a ship at the free surface of an inviscid incompressible fluid is analyzed. Added mass and damping are determined using finite element techniques. Iso-parametric elements with curved boundaries allow accurate representation of the hull shape. A computer program is developed and results are found to agree closely with previously obtained theoretical and experimental results.

14. KEY WORDS	LINK A		LINK B		LINK C	
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